# MATH 142 Practice Exam (a) for Midterm II

**1. (18 points)** Let $R$ the the region bounded by the curve $y = x^4$ and the line $y = 8x$ . Set up, but do not evaluate,

(a) an integral for the area of $R$ using vertical rectangles, i.e. integrate with respect to $x$.

(b) an integral for the area of $R$ using horizontal rectangles, i.e. integrate with respect to $y$.

(c) Calculate the area of $R$ by evaluating either the integral in (a) or (b).

**2. (21 points)** Consider the region $Q$ bounded by the curve $y = x - x^2$ and the $x$-axis. Set up, but do not evaluate, integrals for the following volumes:

(a) The solid obtained by rotating $Q$ about the $y$-axis. For this use the cylindrical shell method.

(b) The solid obtained by rotating $Q$ about the line $x = 3$.

(c) The solid obtained by rotating $Q$ about the $x$-axis.

**3. (13 points)** A trough is 3 meters long, 1 meter wide, and 1 meter deep. The vertical cross section of the trough parallel to an end is shaped like an isoceles triangle (with height 1 meter, and base, on top, of length 1 meter). The trough is 3/4 full of water (i.e. filled to within $\frac{1}{4}$m of the top). Recall the density of water is 1000 kg/$m^3$.

(a) Set up an integral for the amount of work in joules required to empty the trough by pumping the water over the top. (Note: Use $g = 9.8m/s^2$ as the acceleration due to gravity.)

(b) Evaluate the expression in part (a).

**4. (36 points)** Evaluate the following indefinite integrals:

(a) $\int x^2(1-x^3)^9 dx =$

(b) $\int \frac{dx}{x^2\sqrt{x^2-9}} =$

(c) $\int \frac{\sin(x)dx}{\sqrt{1-\cos(x)}} =$

(d) $\int x\ln(x)dx =$

(e) $\int x^2 e^{-x} dx =$

(f) $\int \sin^2(x)\cos^3(x)dx =$

**5. (12 points)** Evaluate the following definite integrals.

(a) $\int_{-2}^{1} x(x+3)^{1/2} dx =$

(b) $\int_{0}^{\pi/2} x\sin(x)dx =$