MATH 142

MIDTERM EXAM I

February 17, 2000

NAME (please print legibly): _____

Your Soc.Sec.Number _____

E-mail address: _____

Circle your Instructor's Name: Gonek McDowall Pizer

- **No calculators are allowed on this exam.**
- **Please show all your work. You may not receive full credit for a correct answer if there is no work shown.**
- **Please put your final answer in the boxes provided**

QUESTION	VALUE	SCORE
1	10	
2	10	
3	7	
4	7	
5	15	
6	9	
7	12	
8	6	
9	9	
10	15	
TOTAL	100	

1. **(10pts)** Consider the function $f(x) = \dfrac{x}{x-1}$.

   (a) Find the vertical asymptote(s) of $f(x)$.

   (b) Find the horizontal asymptote(s) of $f(x)$.

   (c) On what interval(s) is $f(x)$ increasing?

   (d) On what interval(s) is $f(x)$ decreasing?

   (e) On what interval(s) is $f(x)$ concave up?

   (f) On what interval(s) is $f(x)$ concave down?

   (g) Sketch the graph of $f(x)$.

2. **(10pts)** A cardboard box is to be constructed with a square base, a volume of 32 in^3, and an open top (see the figure below).

   (a) What is the **surface area** of the box, as a function of the variable x (see figure)?

   (b) Find the dimensions x and y (see figure) which give the minimal surface area.

3. **(7pts)** Two positive numbers x and y sum to 60. Find the value of y that makes $x\ y^2$ as large as possible. Circle the correct answer:

   A) 35 B) 36 C) 37 D) 38 E) 39

   F) 40 G) 41 H) 42 I) 43

4. **(7pts)** Use Newton's Method with the initial approximation $x_1 = 1.5$ to find x_2, the second approximation to a root of the equation $x^2 - 2 = 0$. Circle the correct answer.

A) $\frac{19}{13}$ B) $\frac{20}{13}$ C) $\frac{19}{12}$ D) $\frac{23}{15}$

E) $\frac{26}{15}$ F) $\frac{17}{12}$ G) $\frac{23}{14}$ H) $\frac{19}{14}$

5. **(15pts)** Find the most general **antiderivative** of the following functions:

(a) $f(x) = x^{\frac{2}{3}} + x^{-\frac{2}{3}}$

(b) $f(x) = e^x + \sqrt{x}$

(c) $f(x) = 2\sin(x) + 3\cos(x)$

(d) $f(x) = \dfrac{x^2 + x + 1}{x}$

(e) $f(x) = \sec^2(x)$

6. **(9pts)** For each of the following, find a function $f(x)$ satisfying:

(a) $f'(x) = \sin(x), \quad f(0) = 5.$

(b) $f''(x) = 20x^3 - 10, \quad f(1) = 1, \quad f'(1) = -5.$

(c) $f''(x) = x, \quad f(0) = 1, \quad f(1) = 2.$

7. **(12pts)** A bee flies in a straight line toward a flower, with an acceleration of $a(t) = 4\ ft/sec^2$. Let $s(t)$ be the bee's position at time t and $v(t)$ its velocity. If $s(2) = 18\ ft$ and $v(2) = 12\ ft/sec$:

(a) Find $v(t)$.

(b) Find $s(t)$.

(c) When is the bee's velocity equal to 32 ft/sec?

(d) How long does it take the bee to reach the flower if the flower is 50 ft from where the bee starts?

8. **(6pts)** Use the **midpoint rule** with $n = 4$ to obtain an approximation for $\int_0^4 x^2 dx$.

9. **(9pts)** Find the derivative of each of the following:

(a) $g(x) = \int_0^x \sqrt{1+2t}\, dt.$

(b) $g(x) = \int_x^2 \sin(t)\, dt.$

(c) $g(x) = \int_0^{x^2} \cos(t)\, dt.$

10. **(15pts)** Evaluate each of the following:

(a) $\int_1^2 \left(3x^2 + 2x + 5\right) dx.$

(b) $\int_1^2 x^{-2}\, dx.$

(c) $\int_0^{\frac{\pi}{2}} \sin(t)\, dt.$

(d) $\int_0^1 e^x\, dx.$

(e) $\int_0^1 \frac{1}{1+x^2}\, dx.$