MATH 142

EXAM I

October 12, 1999

1. (12 points) Find the absolute maximum and absolute minimum of $f(x) = x^4 - 4x^2 + 3$

(a) on $[0, 2]$

Abs. max is at $x =$.

Abs. min is at $x =$.

(b) on $[-1, 1]$

Abs. max is at $x =$.

Abs. min is at $x =$.

NOTE: If an absolute maximum occurs at more than one point, then you just need to give one of those x-coordinates.

2. (20 points) Suppose given a function $f(x)$, and its derivatives $f'(x)$ and $f''(x)$:

$$f(x) = \frac{2x^2}{x^2 - 1} \qquad f'(x) = \frac{-4x}{(x^2 - 1)^2} \qquad f''(x) = \frac{4(3x^2 + 1)}{(x^2 - 1)^3}$$

(a) What is the domain of f?

(b) What are the horizontal and vertical asymptotes of f?

(c) Find the intervals on which f is increasing and the intervals on which f is decreasing.

(d) Find the intervals on which f is concave up and the intervals on which f is concave down.

(e) Sketch the graph of f.

3. (15 points) The graph below is that of the derivative of a continuous function f.

(a) At what values of x does f have a local maximum? a local minimum?

(b) At what values of x does f have a point of inflection (if any)?

(c) Assuming that $f(0) = 0$, sketch a graph of f.

4. (20 points) Find the limit, if it exists. If your answer is $+\infty$ or $-\infty$, be sure to state which one it is.

(a) $\lim_{x\to\infty} \dfrac{(\ln x)^2}{x}$

(b) $\lim_{x\to 0} \dfrac{\sin x}{e^x}$

(c) $\lim_{x\to-\infty} xe^{x^2}$

4. (continued)

(d) $\lim_{x\to 0}(\csc x - \cot x)$

(e) $\lim_{x\to 0^+} x^{\sin x}$

5. (15 points) A rectangular storage container with an open top is to have a volume of 16 m^3. The length of its base is twice the width. Find the dimensions of the container that use the least amount of material. Verify that this is the minimum.

ANSWER: $\qquad\qquad \times \qquad\qquad \times$

6. (12 points) An object is thrown up in the air starting from a height of 80 feet with an initial velocity of 8 ft/second. (The acceleration due to gravity is -32 ft/(sec)^2.)

(a) Find a general formula for the velocity $v(t)$ at time t.

(b) Find a general formula for the height $s(t)$ at time t.

(c) When does the object reach its highest point?

(d) When does the object hit the ground?

7. (6 points) Find the most general form of $f(x)$ if

$$f'(x) = 3e^x + 4\sin x + x\sqrt{x}\,.$$