Mth 142 Practice Final

PART I

1 Evaluate the following integrals.

$$\int_0^4 \frac{1}{\sqrt{x^2+16}}\,dx \quad , \int x\,e^{2x}\,dx \quad , \int \frac{4x}{\sqrt{x^2+16}}\,dx \quad , \int_1^2 x^3\,e^{x^4}\,dx \quad ,$$

$$\int_1^e \frac{\sqrt{\ln x}}{x}\,dx \quad , \int \sin^4 x\,dx \quad , \int_0^{\pi/2} \sin^3 t\cos^2 t\,dt \quad , \int_1^e x^3 \ln x\,dx$$

2. A rectangular swimming pool is 50 m long, 15 m wide and 3 m deep. The depth of the water is 2 m. How much work is required to pump all of the water out over the top? Use $\rho = 1000$ kg/m^3 for the density of water and $g = 10$ m/s^2 for the acceleration of gravity.

3. Find the volume of the solid obtained by rotating the region bounded by $y = \sqrt{x}$, $y = 0$ and $x + y = 2$ about the x-axis.

4. Find the area of the region enclosed by $y = x^3$ and $y = 2x - x^2$.

5. Exercise 55, page 399 of the textbook.

6. Let $I = \int_1^5 e^{-x^2}\,dx$ and let L, R, M and T be the left end-point, right end-point, midpoint and trapezoid sums with $n = 4$, approximating the integral I. Arrange, without computing them, the numbers I, L, M, R, T in increasing order.

PART II

7. A function is known to have the following values:

x	0	2	4	6	8
f(x)	1	4	5	5	7

Use Simpson's rule to approximate $\int_0^8 f(x)\,dx$.

8. Evaluate the following integrals.

$$\int_8^{11} \frac{4x-4}{(x-5)(x+1)}\,dx \quad , \int \frac{x+2}{x(x^2+1)}\,dx$$

9. Determine whether the integral is convergent or divergent. Evaluate in case the integral is convergent.

$$\int_1^\infty \frac{1}{1+x^2}\,dx \quad , \int_0^\infty \frac{1}{(x-1)^2}\,dx \quad , \int_0^\infty e^{2x}\,dx \quad , \int_0^1 \frac{\ln x}{\sqrt{x}}\,dx \quad , \int_1^\infty xe^{-2x}\,dx$$

10. Set up, but do not evaluate, the integral needed to find the area of the surface obtained by rotating the curve about the specified axis.

(a) $y = \sqrt{x} \quad 0 \le x \le 4$ about the x-axis;

(b) $y = \sqrt[3]{x}\,, \quad 0 \le x \le 1$ about the y-axis.

11. Find the length of the curve $y = \frac{2}{3}x^{3/2}\,, \qquad 0 \le x \le 8.$