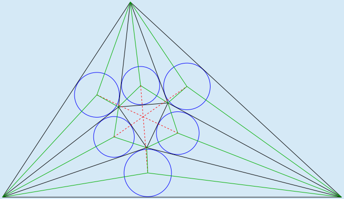
Concurrencies in Morley's Triangle



Maria Nogin
(joint work with Larry Cusick at CSU Fresno)
maria.nogin@csuci.edu

Outline

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Classical Facts
- Morley's Theorem
- Concurrencies

New concurrencies

Proofs

Open questions

Outline

- Classical Facts
  - Morley's Theorem
  - Concurrencies
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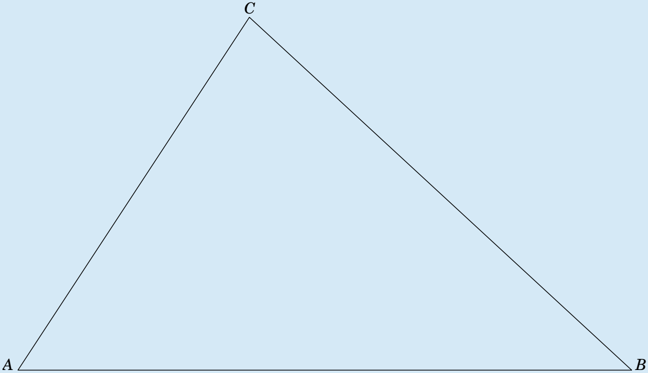
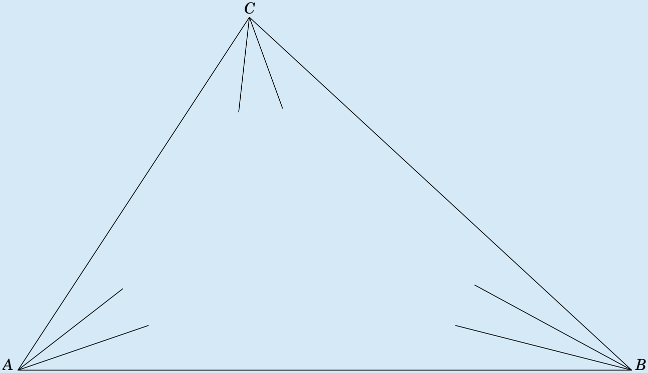
Morley's Theorem



Morley's Theorem



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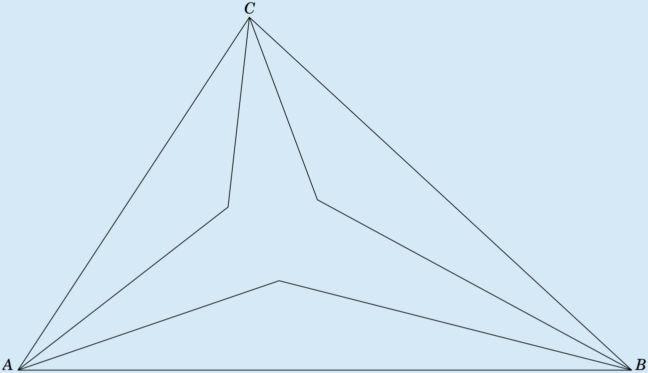

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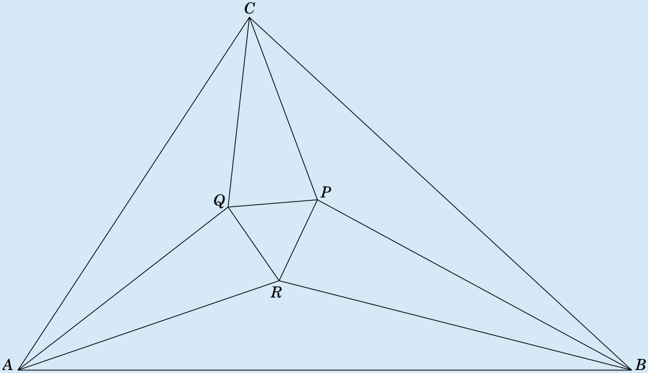
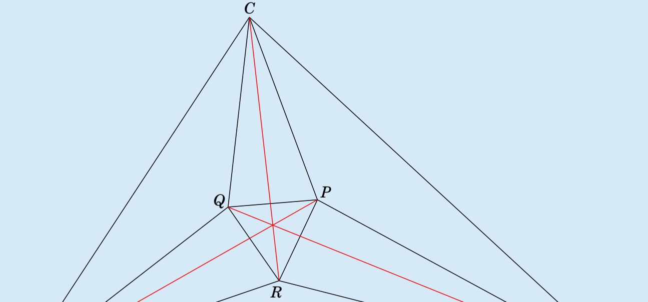


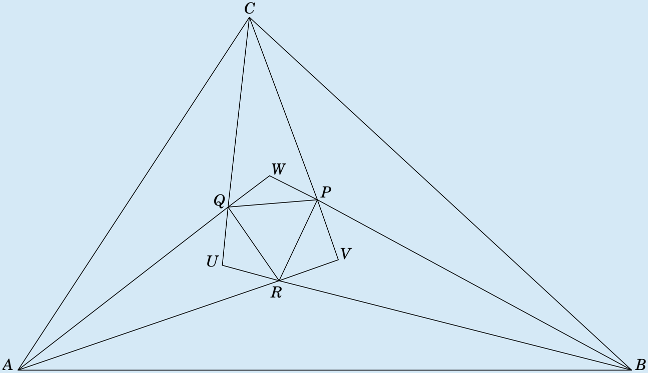
Triangle PQR is equilateral.

Classical concurrencies



The following line segments are concurrent:
AP, BQ, CR

Classical concurrencies



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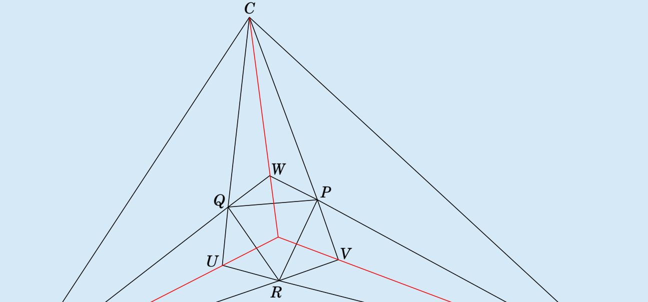

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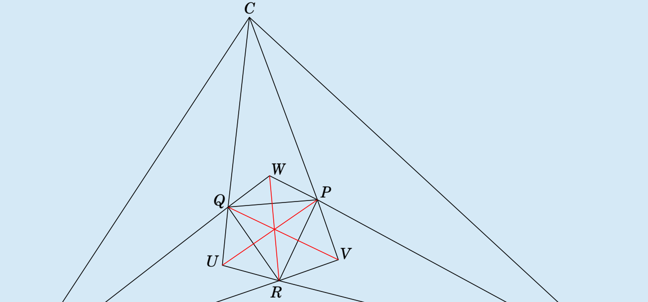
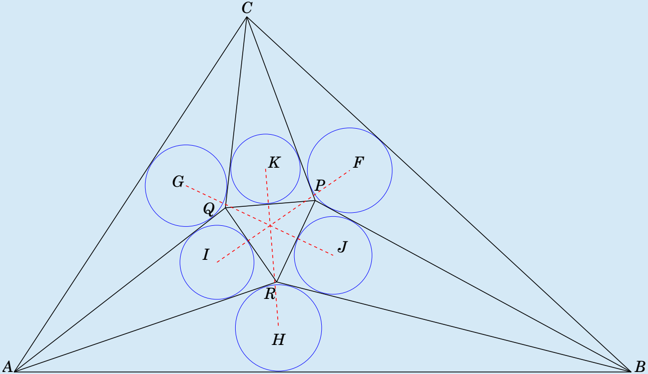


The following line segments are concurrent:

AP, BQ, CR $\qquad$ AU, BV, CW $\qquad$ PU, QV, RW

Larry Cusick's Conjecture



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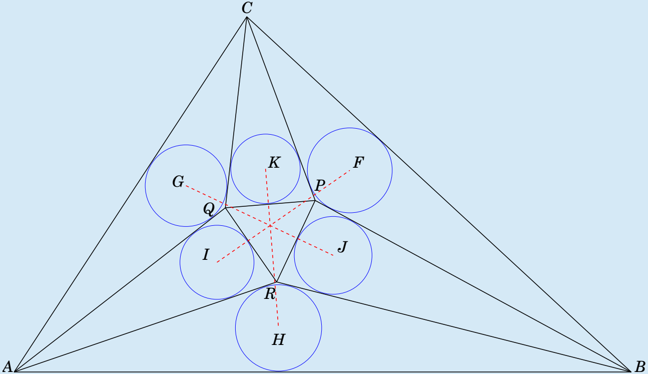

$A(0,0)$
$B(1,0)$
$C(0.377, 0.576)$

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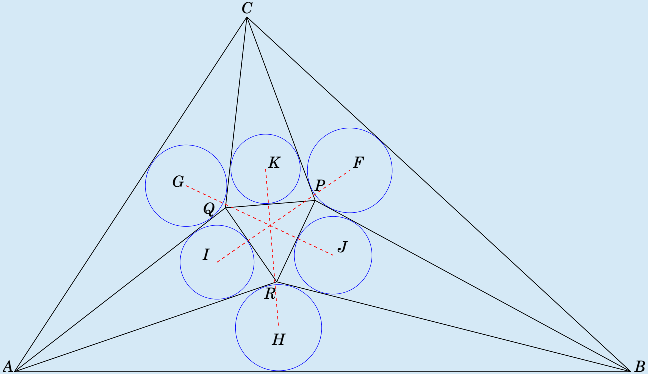

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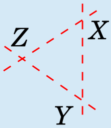

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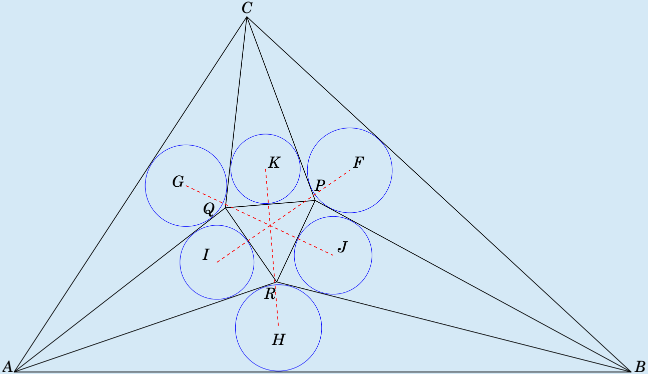

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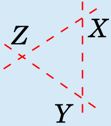
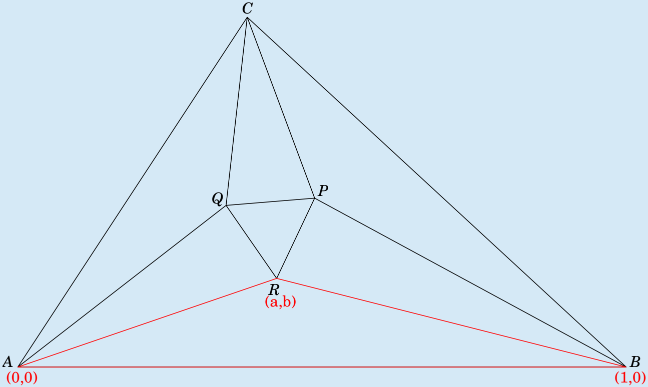


$X(0.414992, 0.237319)$
$Y(0.414992, 0.237314)$
$Z(0.414987, 0.237316)$

New construction – no trisecting!



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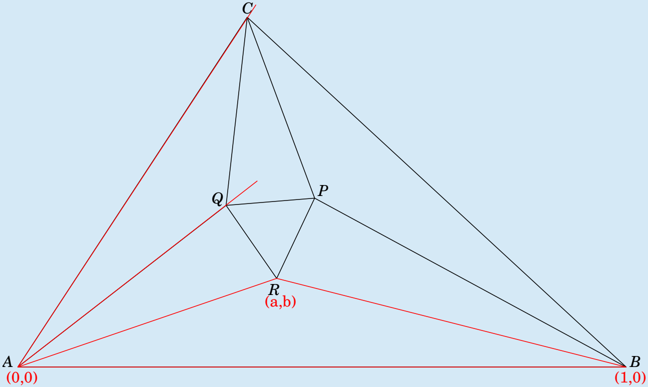

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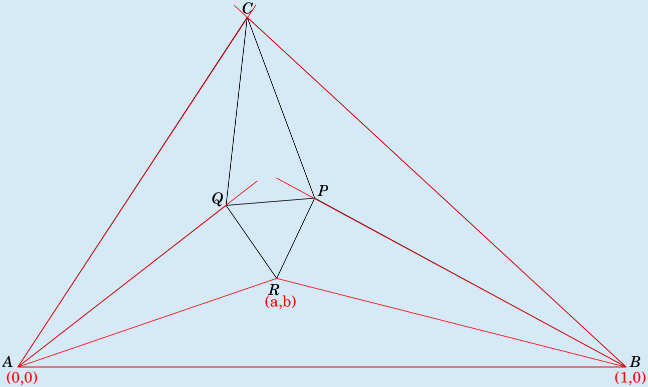

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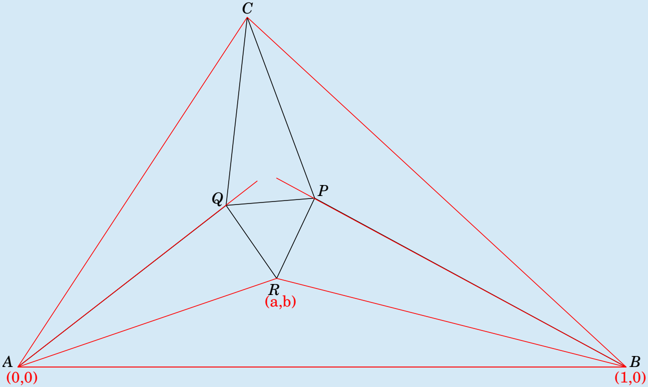

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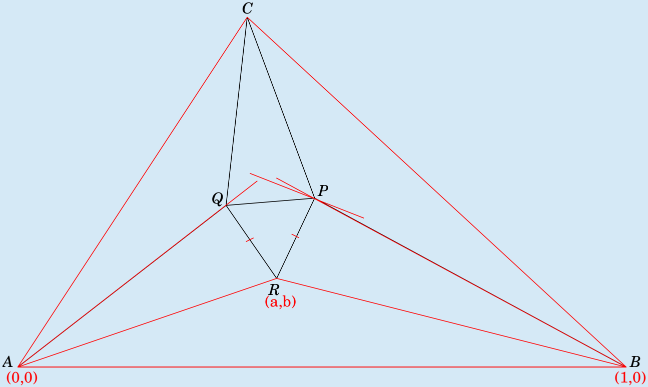

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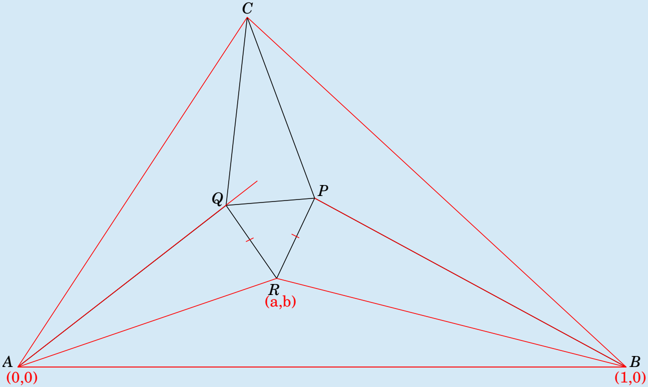

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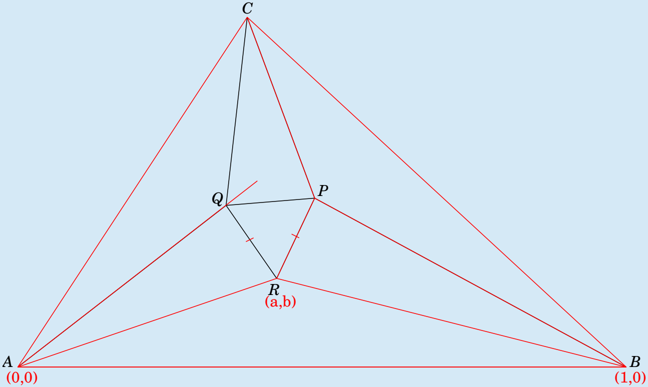

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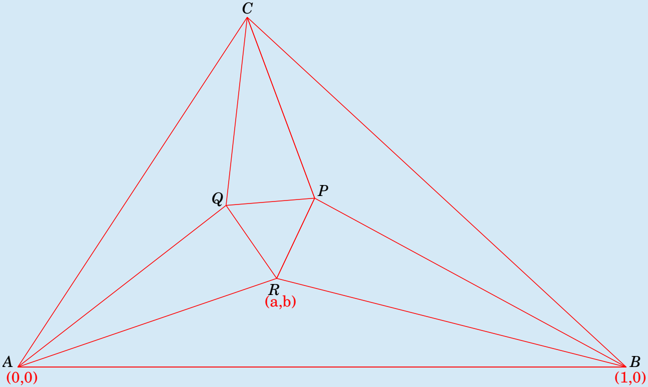
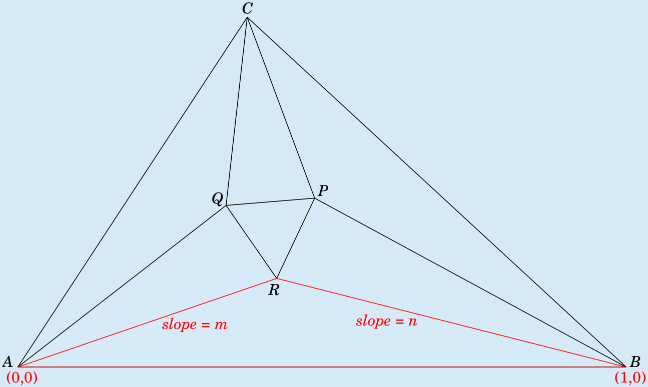


Another approach – using slopes



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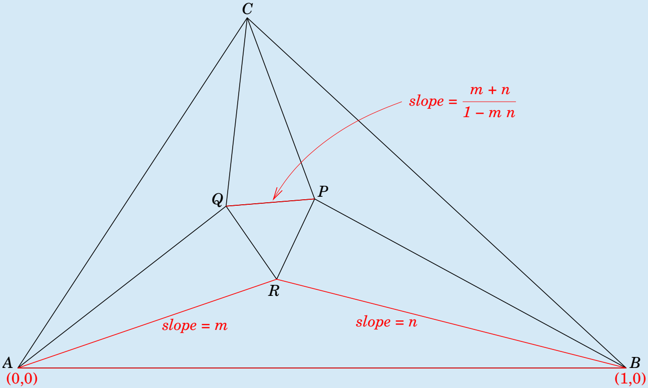

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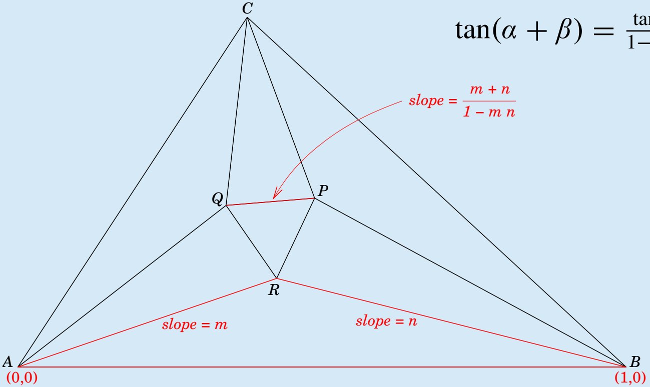
$$\tan(\alpha+\beta) = \frac{\tan(\alpha)+\tan(\beta)}{1-\tan(\alpha)\tan(\beta)}$$



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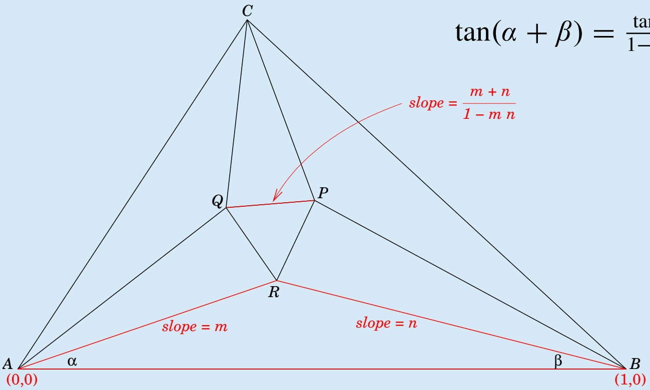

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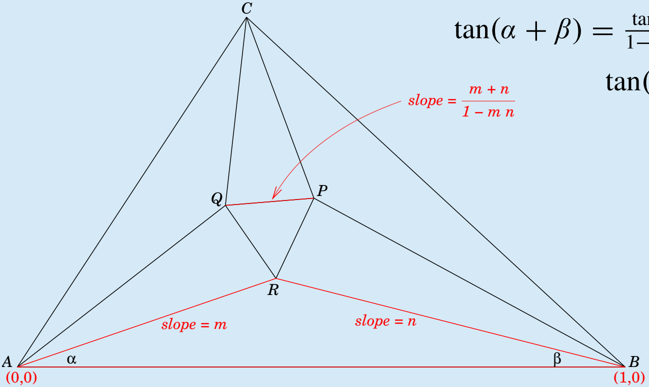

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$$\tan(\alpha) = m$$

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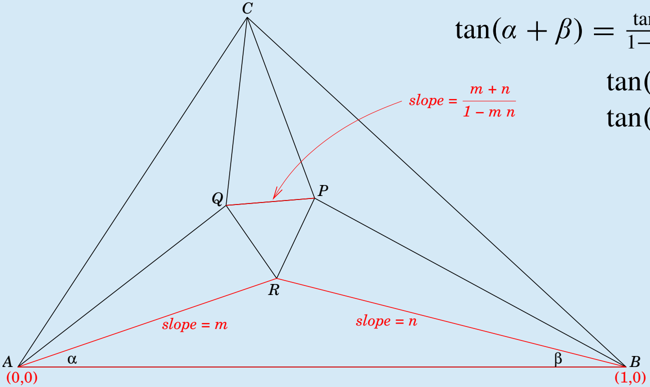

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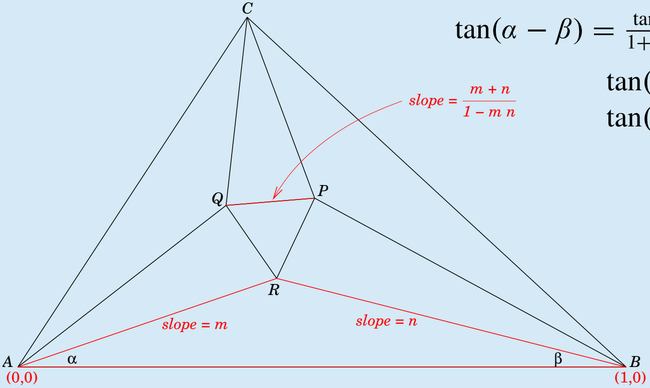

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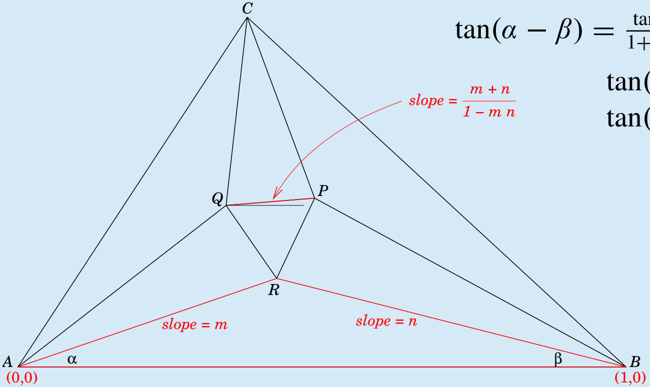

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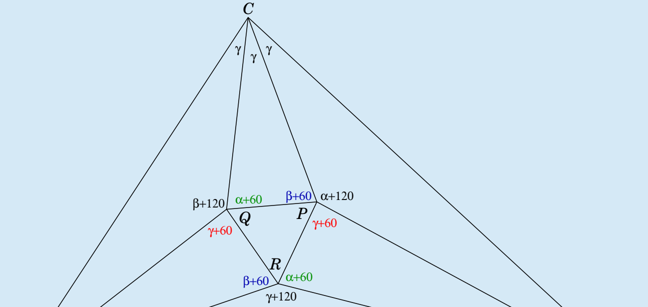
Angles



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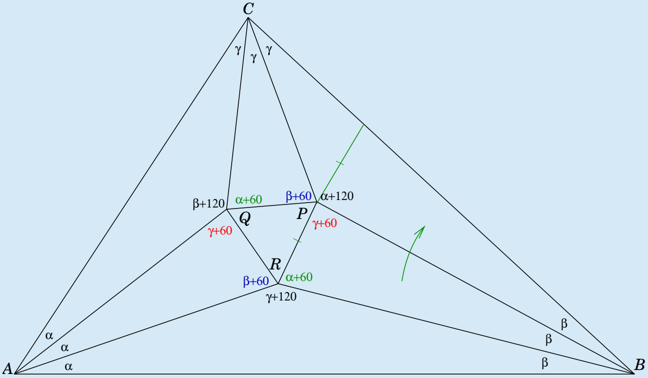

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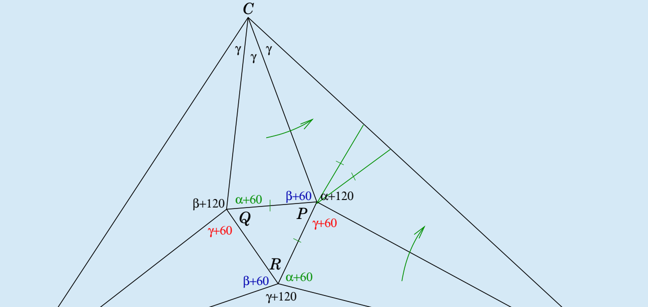

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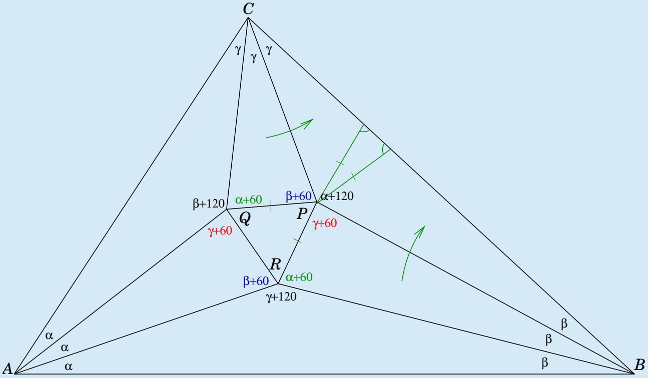

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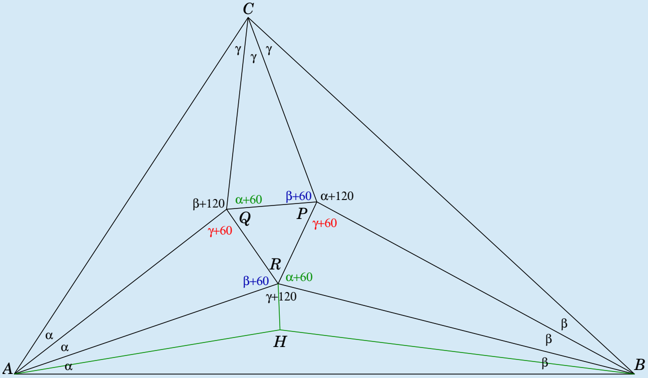


Improved calculations

xA = 0; yA = 0;

xB = 1; yB = 0;

s = 3/7; t = 1/14;

xH = s; yH = t;

slopeAH = (yH-yA) / (xH-xA)

$\frac{1}{6}$

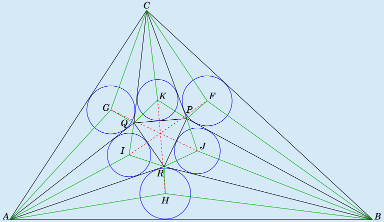
slopeBH = (yH-yB) / (xH-xB)

$-\frac{1}{8}$

slopeAR = 2 * slopeAH / (1 - slopeAH^ 2)

$\frac{12}{35}$

$$\tan(2\alpha) = \frac{2\tan\alpha}{1-\tan^2\alpha}$$



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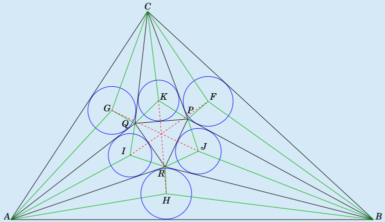

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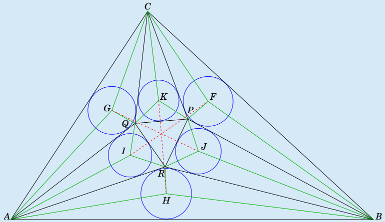

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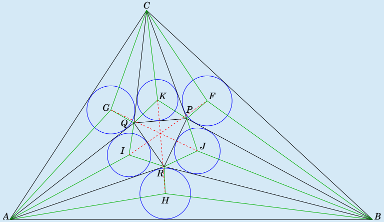

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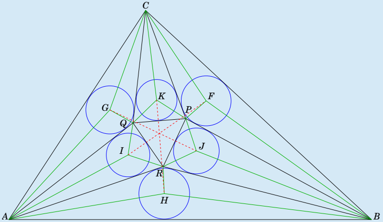

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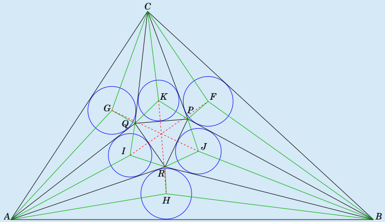

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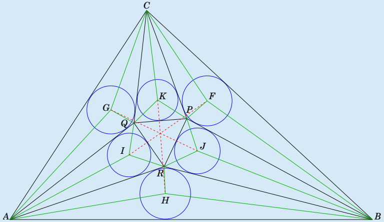

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slopeBR = 2 * slopeBH / (1 - slope BH^ 2)

$-\frac{16}{63}$

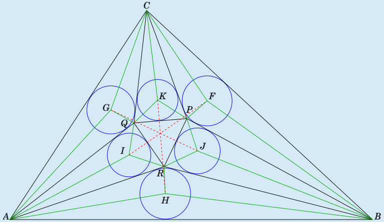
a = - slopeBR/(slopeAR-slopeBR)

$\frac{20}{47}$

b = slopeAR * a

$\frac{48}{329}$

xR = a; yR = b;



slopeAQ = 2 * slopeAR/(1-slopeAR^ 2)

$\frac{840}{1081}$

slopeBP = 2 * slopeBR/(1-slopeBR^ 2)

$-\frac{2016}{3713}$

slopeAC = (slopeAQ+slopeAR)/(1-slopeAQ*slopeAR)

$\frac{42372}{27755}$

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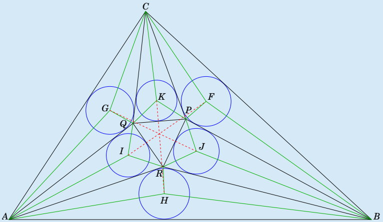

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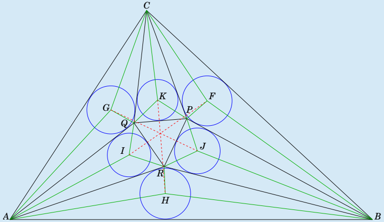

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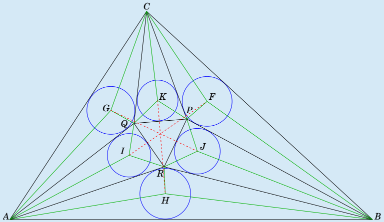

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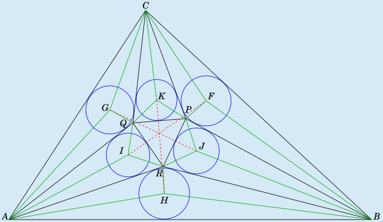


slopeBC = (slopeBP+slopeBR)/(1-slopeBP*slopeBR)

$$-\frac{186416}{201663}$$

...

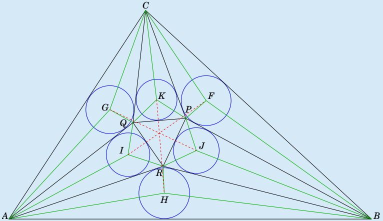
xC = -slopeBC/(slopeAC - slopeBC)

$$\frac{184784860}{489958597}$$

yC = slopeAC * xC

$$\frac{1974704688}{3429710179}$$

...



xQ = (-slopeRQ * a + b) / (slopeAQ - slopeRQ)

$$\frac{92(359401-50700\sqrt{3})}{72972557}$$

yQ = slopeAQ * xQ

$$\frac{480(359401-50700\sqrt{3})}{489958597}$$

...

slopeBC = (slopeBP+slopeBR)/(1-slopeBP*slopeBR)

$-\frac{186416}{201663}$

...

xC = -slopeBC/(slopeAC - slopeBC)

$\frac{184784860}{489958597}$

yC = slopeAC * xC

$\frac{1974704688}{3429710179}$

...

xQ = (-slopeRQ * a + b) / (slopeAQ - slopeRQ)

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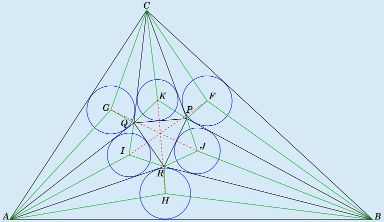

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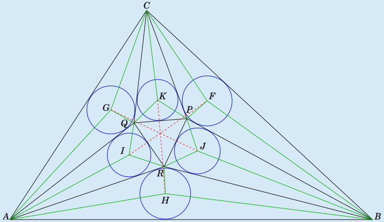

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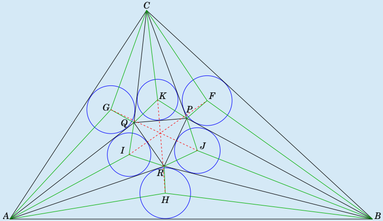

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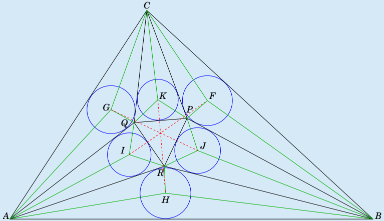

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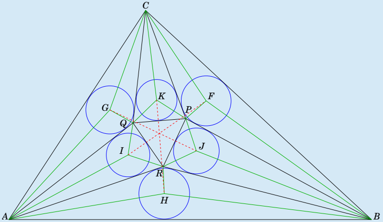

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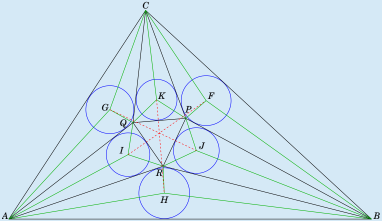

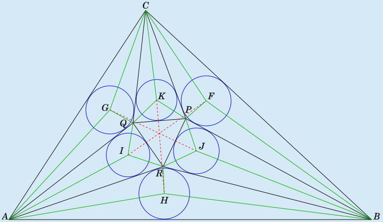

Point X:

Solve[{(yX-yF) / (xX-xF) == (yI-yF) / (xI-xF),
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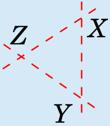

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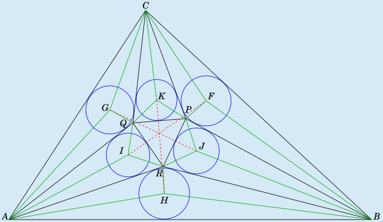

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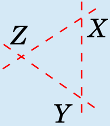

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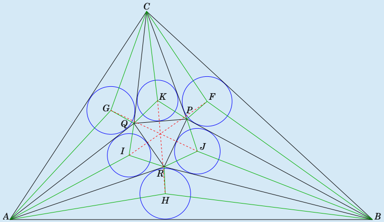

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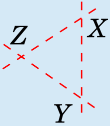

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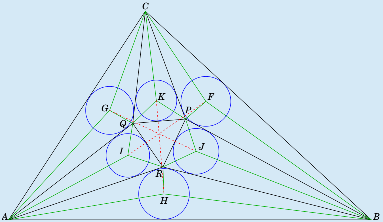

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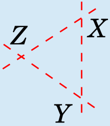

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Point X:

$$xX = \frac{4(224985396519102569937662510-217405012330203822072753831\sqrt{3})}{533309(5693704-530291\sqrt{3})(591745618086307-672855655355026\sqrt{3})}$$

$$yX = \frac{5136(37093246035964629-29577692233425689\sqrt{3})}{533309(591745618086307-672855655355026\sqrt{3})}$$

xX=0.414992, yX=0.237319

Point Y:

$$xY = \frac{4(995191104328017002503243868 79-901269081544109739081572612 54\sqrt{3})}{533309(2313189467-314659968\sqrt{3})(574817123835459-665802139259414\sqrt{3})}$$

$$yY = \frac{27504(6801737015710231-5463593276362747\sqrt{3})}{533309(574817123835459-665802139259414\sqrt{3})}$$

xY=0.414992, yY=0.237314

Point Z:

$$xZ = \frac{4(6791707397634201236043864 1-6154094833168959782661268 0\sqrt{3})}{533309(5693704-530291\sqrt{3})(182504369382776-189881817047527\sqrt{3})}$$

$$yZ = \frac{980976(5792592277 1199-4434715861 2077\sqrt{3})}{533309(182504369382776-189881817047527\sqrt{3})}$$

xW=0.414987, yW=0.237316

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Another conjecture

Are lines AF, BG, and CH concurrent?

xA = 0; yA = 0;

xB = 1; yB = 0;

xH = s; yH = t;

slopeAH = (yH-yA) / (xH-xA)

$\frac{t}{s}$

slopeBH = (yH-yB) / (xH-xB)

$\frac{t}{s-1}$

slopeAR = 2 * slopeAH / (1 - slopeAH^ 2)

$\frac{2st}{s^2-t^2}$

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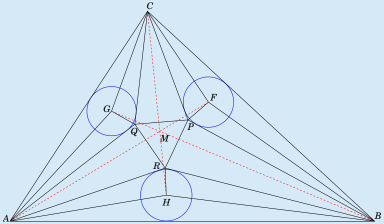

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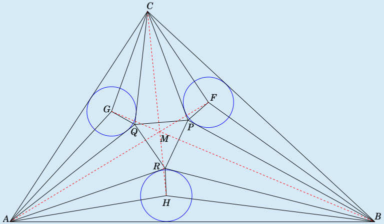

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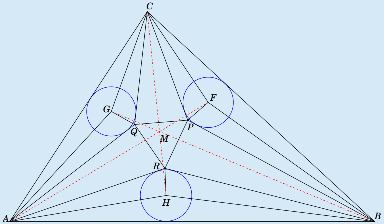

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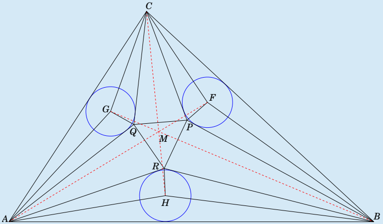

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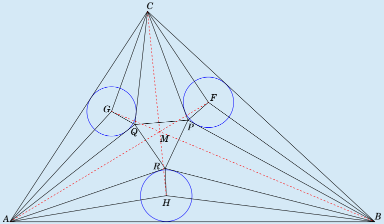

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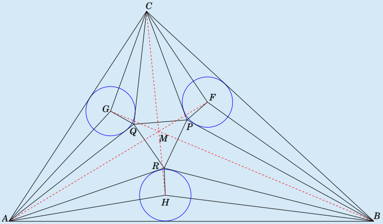

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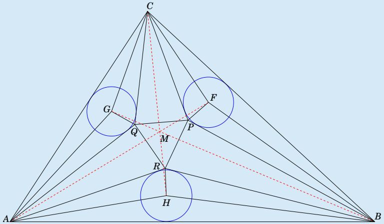

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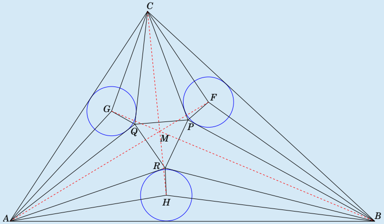

slopeCH = Simplify[(yH-yC)/(xH-xC)]

$-15(s-1)^5s^5+5(s-1)^3s^3(10+27(s-1)s)t^2-(s-1)s(15+2(s-1)s(51+103(s-1)s))t^4+3(1+10(s-1)s(1+5(s-1)s))t^6-(10+3(s-1)s)t^8+3t^{10})/(35(s-1)^4s^4(2s-1)t-42(s-1)^2s^2(2s(2+s(2s-3))-1)t^3+3(2s-1)(1+2(s-1)s(2+19(s-1)s))t^5-10(2s(2+s(2s-3))-1)t^7+3(2s-1)t^9)$

slopeCM = Simplify[(yM-yC)/(xM-xC)]

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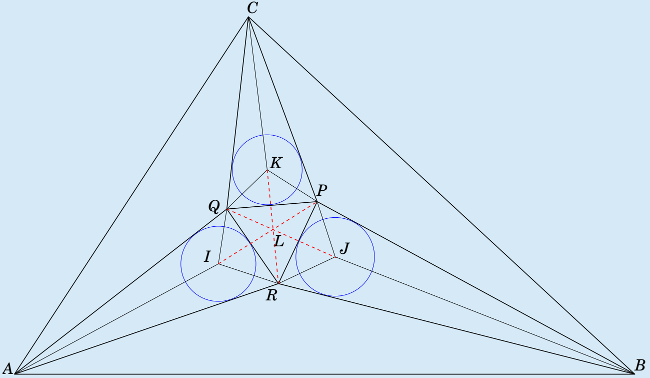
slopeCM = Simplify[(yM-yC)/(xM-xC)]

$-15(s-1)^5 s^5 + 5(s-1)^3 s^3 (10 + 27(s-1)s)t^2 - (s-1)s(15 + 2(s-1)s(51 + 103(s-1)s))t^4 + 3(1 + 10(s-1)s(1 + 5(s-1)s))t^6 - (10 + 3(s-1)s)t^8 + 3t^{10})/(35(s-1)^4 s^4 (2s-1)t - 42(s-1)^2 s^2 (2s(2 + s(2s-3)) - 1)t^3 + 3(2s-1)(1 + 2(s-1)s(2 + 19(s-1)s))t^5 - 10(2s(2 + s(2s-3)) - 1)t^7 + 3(2s-1)t^9)$

These slopes are identical, so the lines are concurrent!

Another one

Are lines PI, QJ, and RK concurrent?



slopeRK = Simplify[(yK-yR)/(xK-xR)]

$$(6s^5 - 2s^6 + 2st^2(-2 + 3\sqrt{3}t + 5t^2) + s^4(-6 + \sqrt{3}t + 20t^2) - 2s^3(-1 + \sqrt{3} + 20t^2) + t^3(-\sqrt{3} - 2t + \sqrt{3}t^2) + s^2t(\sqrt{3} + 24t - 6\sqrt{3}t^2 - 10t^3))/((2s - 1)t(-10s^3 + 5s^4 + 2st(\sqrt{3} + 5t) + s^2(5 - 2\sqrt{3}t - 10t^2) + t^2(-1 + 2\sqrt{3}t + t^2)))$$

slopeKL = Simplify[(yL-yK)/(xL-xK)]

$$(-30s^{21} + 30s^{20}(10 + \sqrt{3}t) + s^{19}(-1350 - 143\sqrt{3}t + 378t^2) - 3s^{18}(-1200 + t(21\sqrt{3} + t(1291 + 110\sqrt{3}t))) + s^{17}(-6300 + t(2052\sqrt{3} + t(16884 + 1073\sqrt{3} - 600t^2))) + s^{16}(7560 + t(-6888\sqrt{3} + t(-41004 + t(4259\sqrt{3} + 27t(361 + 8\sqrt{3})))) + 2s^{15}(-3150 + t(6111\sqrt{3} - 2t(-15057 + t(8054\sqrt{3} + 3t(3734 - 119\sqrt{3}t + 258t^2))))) + 2s^{14}(1800 + t(-6741\sqrt{3} + t(-26523 + 2t(21357\sqrt{3} + t(22122 - 6527\sqrt{3}t + 4491t^2 + 534\sqrt{3}t^3))))) - 3t^{11}(3t^2 - 1)(9\sqrt{3} + t(9 + t(-33\sqrt{3} + t(15 + t(35\sqrt{3} + t(-61 + 13\sqrt{3}t - 3t^2)))))) - \ldots$$ 4 more pages!

slopeRK = Simplify[(yK-yR)/(xK-xR)]

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slopeRK = Simplify[(yK-yR)/(xK-xR)]

$$(6s^5-2s^6+2st^2(-2+3\sqrt{3}t+5t^2)+s^4(-6+\sqrt{3}t+20t^2)-2s^3(-1+\sqrt{3}+20t^2)+t^3(-\sqrt{3}-2t+\sqrt{3}t^2)+s^2t(\sqrt{3}+24t-6\sqrt{3}t^2-10t^3))/((2s-1)t(-10s^3+5s^4+2st(\sqrt{3}+5t)+s^2(5-2\sqrt{3}t-10t^2)+t^2(-1+2\sqrt{3}t+t^2)))$$

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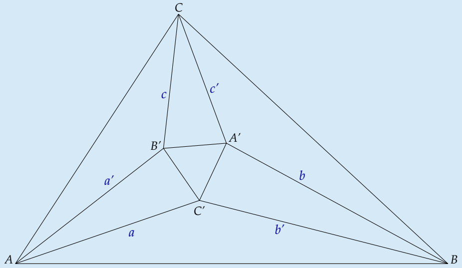
Together[slopeRK-slopeKL]

0

These slopes are equal, so the lines are concurrent!

Definition Given $\angle A$, lines a and a' through A are called isogonal if they are reflections of each other in the bisector of $\angle A$.



Theorem Let pairs of lines a and a', b and b', c and c' be isogonal at vertices A, B, and, C, respectively.
Let $a \cap b' = C'$, $b \cap c' = A'$, and $c \cap a' = B'$.
Then the lines AA', BB', and CC' are concurrent.

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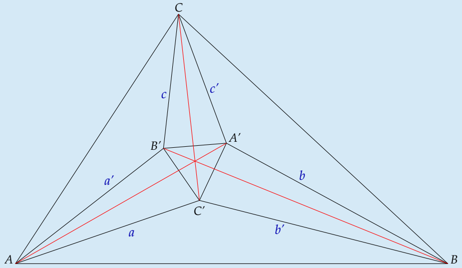

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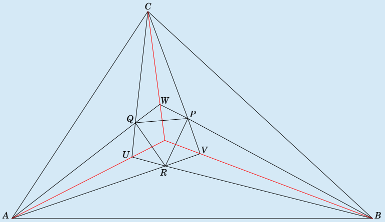
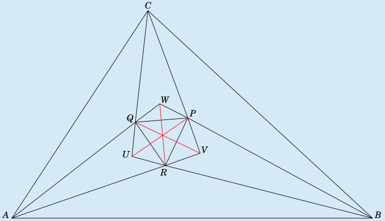
Other classical concurrencies





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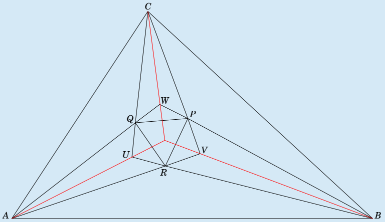
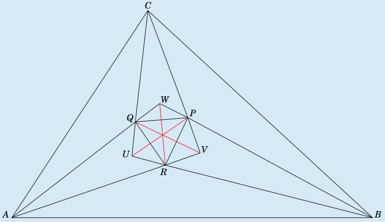
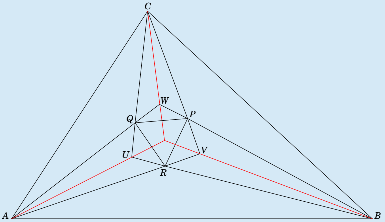




Isogonality argument works!

Other classical concurrencies



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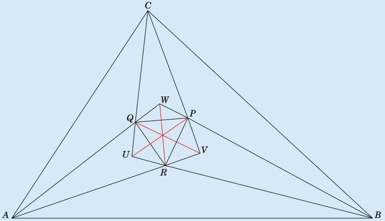
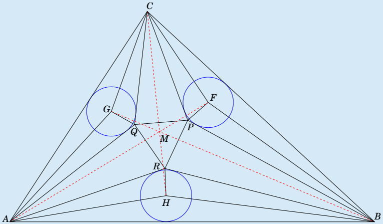
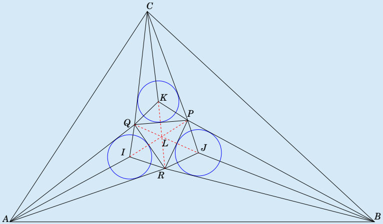


Not so easy!

Proof of the new concurrencies





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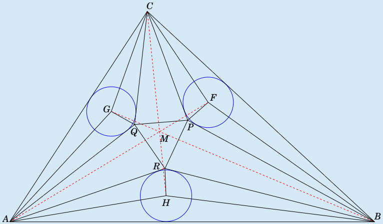

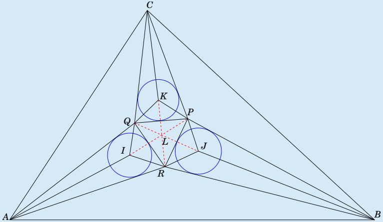

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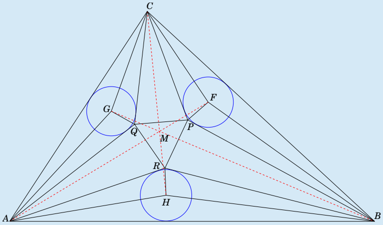

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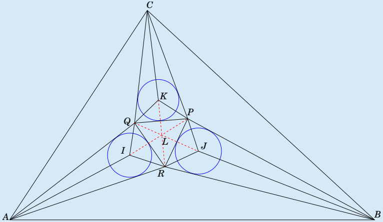

Not so easy!

Open questions

Open questions (= possible projects)

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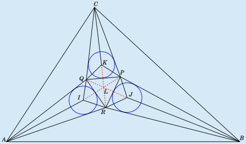
1. Find an elementary geometry proof of this concurrency:



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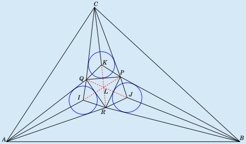
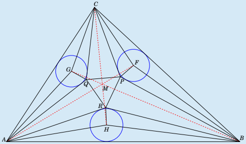
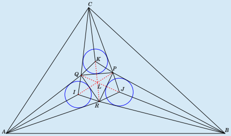


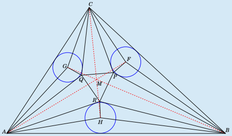
2. Find the barycentric and/or trilinear coordinates of both intersection points.



Open questions (= possible projects)

3 Are these intersection points same as some known triangle centers?





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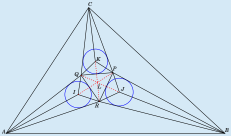

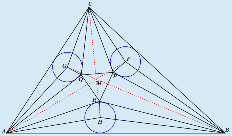

4 Any other concurrencies?