Math Field Day

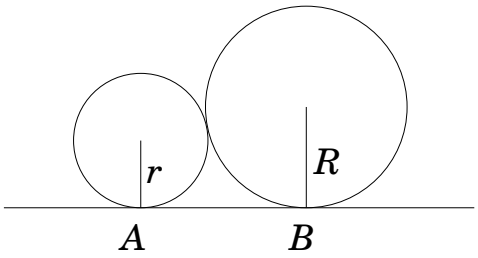
Leap Frog Relay sample questions - 2

1. Suppose r_1, r_2, r_3 are the three roots to the cubic equation $2002 + 2003x + 2004x^2 + 2005x^3 = 0$. Then $\dfrac{1}{r_1} + \dfrac{1}{r_2} + \dfrac{1}{r_3} =$
   (a) $-\dfrac{2004}{2003}$
   (b) $-\dfrac{2003}{2002}$
   (c) $-\dfrac{2004}{2005}$
   (d) $-\dfrac{2005}{2004}$
   (e) None of these

2. The value of the infinite continued fraction $1 + \dfrac{1}{1 + \frac{3}{1+\frac{1}{1+\frac{3}{1+\ldots}}}}$ is
   (a) $\dfrac{1+\sqrt{5}}{2}$
   (b) $\sqrt{3}$
   (c) $\dfrac{-1+\sqrt{13}}{2}$
   (d) 3
   (e) None of these

3. If $\sin\theta^o = \dfrac{1}{3}$ and $90^o < \theta^o < 180^o$, then $\dfrac{\cos\frac{\theta^o}{2} - \sin\frac{\theta^o}{2}}{\cos\frac{\theta^o}{2} + \sin\frac{\theta^o}{2}} =$
   (a) $\dfrac{-\sqrt{2}}{2}$
   (b) $\dfrac{\sqrt{2}}{2}$
   (c) $\dfrac{-\sqrt{3}}{3}$
   (d) $\dfrac{\sqrt{3}}{3}$
   (e) None of these

4. The two circles pictured are mutually tangent and tangent to the line $\bar{AB}$ at the respective points A and B. Determine the distance AB as a function of r and R.



   (a) $\sqrt{r} + \sqrt{R}$
   (b) $\sqrt{r + R}$
   (c) $\dfrac{r + R}{2}$
   (d) $2\sqrt{rR}$
   (e) None of these

5. Among all real number pairs (x, y) that satisfy $x^2 + x + y^2 + y = 1$, find the largest possible value of $x + y$.
   (a) $\sqrt{2} - 1$
   (b) 1
   (c) $\sqrt{3} - 1$
   (d) $\sqrt{3}$
   (e) None of these

6. The value of the sum $\displaystyle\sum_{k=0}^{2004} \cos \frac{k\pi}{2004}$ lies in the interval
   (a) between -0.5 and 0.5
   (b) less than -0.5
   (c) between 0.5 and 1.5
   (d) more than 1.5
   (e) None of these

7. You select N integers, $x_1, x_2, \ldots, x_N$, at random. What is the smallest value of N that will insure that at least one difference $x_i^2 - x_j^2$, $i \neq j$, is divisible by 5?
   (a) 10
   (b) 5
   (c) 4
   (d) 6
   (e) None of these