

CSU FRESNO MATHEMATICS FIELD DAY
APRIL 16, 2005
MAD HATTER MARATHON B
PART I

1. Convert 432 (base 10) to a base 5 value.
 - (a) 3212_{five}
 - (b) 2312_{five}
 - (c) 432_{five}
 - (d) none of the above

2. Convert 346_{seven} to a base 10 value
 - (a) 181
 - (b) 346
 - (c) 567
 - (d) none of the above

3. The ratio of x to y is 5:3. The ratio of y to z is 9:10. What is the ratio of x to z ?
 - (a) 3:2
 - (b) 2:1
 - (c) 3:1
 - (d) none of the above

4. Joe sold 180 newspapers out of 200. Bill sold 85% of his 260 newspapers. Ron sold 212 newspapers, 80% of those he had. Who sold most newspapers and how many newspapers did he sell?
 - (a) Ron sold 212
 - (b) Bill sold 221
 - (c) Ron and Bill both sold 212
 - (d) none of the above

5. The area of a rectangle is 1363 square centimeters. If the length and width are both prime numbers, what is the perimeter of the rectangle?
 - (a) 681 cm
 - (b) 76 cm
 - (c) 152 cm
 - (d) none of the above

6. What is the sum of the coefficients of $(x - y)^5$?
 - (a) 8
 - (b) 0
 - (c) 12
 - (d) none of the above

7. Simplify: $(a + b)^3 - (a - b)^3$

- (a) $9a^2b + 6b^3$
- (b) $6a^2b + 2b^3$
- (c) $a^2b + 6b^3$
- (d) none of the above

8. Express $x < -8$ or $x > -4$ in terms of an absolute value.

- (a) $|x - 6| > 2$
- (b) $|x + 6| < 2$
- (c) $|x + 6| > 2$
- (d) $|x - 6| < 2$

9. Let $f(x) = 3x - 4$. Find the inverse of f .

- (a) $\frac{x - 4}{3}$
- (b) $\frac{x + 3}{4}$
- (c) $\frac{x + 4}{3}$
- (d) none of the above

10. Carrie's bowling scores for four games are $b + 2$, $b + 3$, $b - 2$, and $b - 1$. What must her score be on her fifth game if her average score is $b + 2$?

- (a) $b - 2$
- (b) $b + 5$
- (c) $b + 8$
- (d) none of the above

11. Write $2.\overline{345}$ as a fraction in lowest terms

- (a) $\frac{129}{55}$
- (b) $\frac{2322}{999}$
- (c) $\frac{21149}{990}$
- (d) none of the above

12. Find x so that $3x - 1$, $5x - 4$, $2x + 8$ are consecutive terms of an arithmetic sequence.

- (a) 2
- (b) 5
- (c) 4
- (d) none of the above

13. A radio station gave away a discount coupon for every twelfth and thirteenth caller. Every twentieth caller received a free concert ticket. Which caller was the first to get both a coupon and a free concert ticket?

- (a) 45th caller
- (b) 20th caller
- (c) 60th caller
- (d) none of the above

14. Determine the value k that makes $25x^2 + kx + 49$ a perfect square trinomial.

- (a) 35
- (b) 140
- (c) 70
- (d) none of the above

15. Find the numbers A , B , C , and D in the following base 6 addition.

$$\begin{array}{rcccc} & 3 & A & B & 3 \\ + & & 2 & 5 & C \\ \hline D & 0 & 0 & 0 & 2 \end{array}$$

- (a) $A = 1, B = 2, C = 3, D = 4$
- (b) $A = 3, B = 0, C = 5, D = 3$
- (c) $A = 3, B = 0, C = 5, D = 4$
- (d) none of the above

16. For what positive value of x is $x + 3$ the reciprocal of $x - 3$?

- (a) $2\sqrt{2}$
- (b) $4\sqrt{2}$
- (c) 8
- (d) none of the above

17. A man travels 2 miles north, 2 miles east, one mile south, one mile west, 3 miles north, and 3 miles east. How far is he from the starting point?

- (a) $2\sqrt{4}$ miles
- (b) 6 miles
- (c) $4\sqrt{2}$ miles
- (d) none of the above

18. Find the seventh term in the expansion of $(A + B)^{10}$.

- (a) $210A^2B^8$
- (b) $-120A^4B^6$
- (c) $120A^2B^8$
- (d) none of the above

19. Simplify $\frac{\log_7 10}{\log_7 1000}$.

- (a) 3
- (b) $-\frac{1}{3}$
- (c) $\frac{1}{3}$
- (d) none of the above

20. Solve for x : $|2x + 1| > 1$

- (a) $-4 < x < 4$
- (b) $x > 4$
- (c) $x < 0$
- (d) none of the above

21. An $8 \text{ mm} \times 8 \text{ mm}$ square has its corners cutoff to make a regular octagon. Find the area of the octagon.

- (a) 20 mm^2
- (b) 100 mm^2
- (c) 40 mm^2
- (d) none of the above

22. Find $4 + 8 + 16 + \dots + 16384$.

- (a) 27670
- (b) 27634
- (c) 32764
- (d) none of the above

23. Find the ratio of A to B if $\frac{2B + 6A}{3A - B} = 10$.

- (a) 1:3
- (b) 3:1
- (c) 1:2
- (d) none of the above

24. What number is doubled when two-thirds of it is added to 36?

- (a) 36
- (b) 2
- (c) 27
- (d) none of the above

25. Simplify $\frac{\frac{x}{x+y} + \frac{y}{x-y}}{\frac{y}{x+y} - \frac{x}{x-y}}$.

- (a) 1
- (b) -1
- (c) $\frac{1}{2}$
- (d) $-\frac{1}{2}$

26. Simplify $(1 - (-i)^{318})^2$.

- (a) 4
- (b) i
- (c) 0
- (d) none of the above

27. The point (a, b) is reflected over the y -axis to the point (c, d) , which is reflected over the x -axis to the point (e, f) . What is $ab - ef$?

- (a) 2
- (b) $2ab$
- (c) 0
- (d) none of the above

28. The first digit of a seven-digit telephone number can never be a 0. How many telephone numbers have increasing digits?

- (a) 28
- (b) 720
- (c) 36
- (d) none of the above

29. Evaluate $\frac{12!}{8!6!4!}$.

(a) $\frac{11}{16}$

(b) $\frac{9}{16}$

(c) $\frac{7}{16}$

(d) $\frac{5}{16}$

30. Simplify $\left(\frac{y^{-4}}{x^{-2}}\right)^{-\frac{1}{4}}$.

(a) $\frac{x}{y}$

(b) $\frac{y}{x}$

(c) $-\frac{x}{y}$

(d) none of the above

31. Evaluate $\frac{3^4 \cdot 3^1 - 3^2}{3^3 + 2}$.

(a) $\frac{230}{29}$

(b) $\frac{231}{29}$

(c) $\frac{232}{29}$

(d) none of the above

32. At Rattlesnake School the teacher - student ratio is 1:30. If the school has 1200 students, how many additional teachers must be hired to reduce the ratio to 1:20?

(a) 15

(b) 10

(c) 20

(d) none of the above

33. Solve $\begin{cases} 2x+3y = 1 \\ x+2y = 3 \end{cases}$.

- (a) $x = 7, y = 5$
- (b) $x = -7, y = -5$
- (c) $x = 7, y = -5$
- (d) none of the above

34. Solve $\frac{1}{x-1} > \frac{1}{3}$.

- (a) $-4 < x < -1$
- (b) $1 < x < 1$
- (c) $x < 1$ or $x > 4$
- (d) none of the above

35. Each side of a square is x cm long. The length of a rectangle is 5 cm longer than the side of the square, and the width of the rectangle is 5 cm shorter than the side of the square. Which one has greater area and by how much?

- (a) square, 125 cm^2
- (b) square, 25 cm^2
- (c) rectangle, 125 cm^2
- (d) rectangle, 25 cm^2

36. A balance scale was in perfect balance when Horace placed a box of candy on one pan of the scale and $\frac{3}{4}$ of the same size candy box together with a $\frac{3}{4}$ -pound weight on the other pan. How much did the full box of candy weigh?

- (a) 4 pounds
- (b) 6 pounds
- (c) 3 pounds
- (d) none of the above

37. An integer between 1 and 1000 (inclusive) is selected at random. Find the probability that the integer is divisible by 5.

- (a) $\frac{1}{10}$
- (b) $\frac{1}{100}$
- (c) $\frac{1}{5}$
- (d) $\frac{1}{1000}$

38. If N and M are positive integers, then $\frac{2N}{M} < \frac{2N+2}{M+1}$ if and only if

- (a) $N < M$
- (b) $M < N$
- (c) $N \leq M$
- (d) $M \leq N$

39. Solve $2x \geq x + 1$.

- (a) $x \leq 1$
- (b) $x \geq 1$
- (c) $x = 1$
- (d) none of the above

40. Evaluate and express $\frac{35,921,000 \times 62 \times 10^3}{3.1 \times 10^5}$ in scientific notation.

- (a) 7.1842×10^6
- (b) 7.1842×10^7
- (c) 7.1842×10^8
- (d) none of the above

MAD HATTER MARATHON B

PART I ANSWERS

- | | |
|-------|-------|
| 1. a | 21. d |
| 2. a | 22. c |
| 3. a | 23. c |
| 4. b | 24. c |
| 5. c | 25. b |
| 6. b | 26. a |
| 7. b | 27. c |
| 8. c | 28. c |
| 9. c | 29. a |
| 10. c | 30. d |
| 11. a | 31. d |
| 12. d | 32. c |
| 13. c | 33. d |
| 14. c | 34. b |
| 15. c | 35. b |
| 16. d | 36. c |
| 17. c | 37. c |
| 18. d | 38. a |
| 19. c | 39. b |
| 20. d | 40. a |

PART II

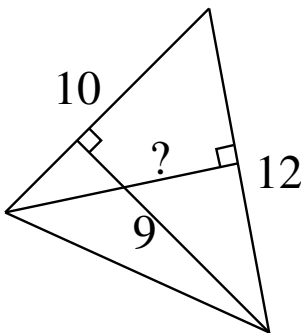
1. Solve for x : $\sqrt{1 + \sqrt{2 + \sqrt{x}}} = 3$.

- (a) 78
- (b) 3844
- (c) 15
- (d) none of the above

2. A bag of 3 apples, 7 oranges, and 11 pears cost \$6.04, while a bag of 2 apples, 5 oranges, and 8 pears costs \$4.31. What is the cost of a bag of fruit consisting of 1 apple, 1 orange and 1 pear?

- (a) \$2.34
- (b) \$1.25
- (c) \$0.88
- (d) none of the above

3. Two sides of a triangle measure 10 cm and 12 cm, and the altitude to the shorter of the two sides measures 9 cm. What is the length of the altitude to the longer side?

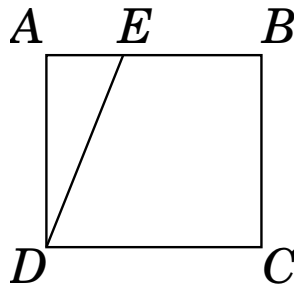


- (a) 7.5 cm
- (b) 11 cm
- (c) 8 cm
- (d) none of the above

4. If the surface area of a sphere is 4π , find its volume.

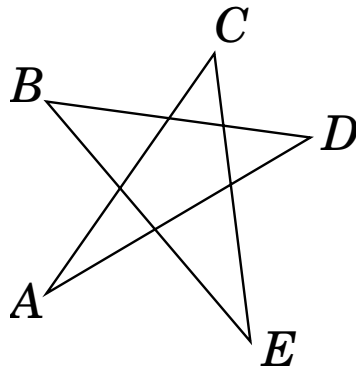
- (a) $\frac{4\pi}{3}$
- (b) $\frac{16\pi}{3}$
- (c) $\frac{3\pi}{4}$
- (d) none of the above

5. In the figure below, $ABCD$ is a rectangle and $DE = DC$. Given $AD = 5$ and $BE = 3$, what is DE ?



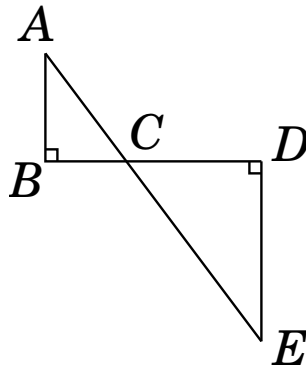
- (a) 2
(b) 17
(c) 8
(d) none of the above
6. Given the expression xy^2 , if the values of x and y are each decreased by 50 percent, what is the percent of decrease in the value of the original expression?
- (a) 119 percent
(b) 57.6 percent
(c) 87.5 percent
(d) none of the above
7. A collection contains \$6.70. There are twice as many dimes as quarters and two more nickels than dimes. Find the number of dimes.
- (a) 26
(b) 12
(c) 24
(d) none of the above
8. If A is 35 percent less than B and if C is 75 percent more than B, approximately what percent of C is A?
- (a) 35%
(b) 37.14%
(c) 2.69%
(d) none of the above

9. Let a , b , c , d , and e be the inner angles of the vertices A , B , C , D and E respectively. What is the value of $a + b + c + d + e$?



- (a) 360 degrees
(b) 180 degrees
(c) cannot be determined
(d) none of the above
10. A bag contains three blue, four red, and three yellow marbles. How many blue marbles must be added to the bag for it to contain 75 percent blue marbles?
- (a) 16
(b) 18
(c) 12
(d) none of the above
11. A car travels a distance of 1742 miles at an average rate of 40 miles per hour and returns at an average rate of 55 miles per hour. What is the average rate for the round trip to the nearest tenth?
- (a) 47.5
(b) 46.3
(c) 48.6
(d) none of the above

12. In the diagram below, $BD = 6$ km, $AB = 3$ km, and $DE = 5$ km. What is the length of AE ?



- (a) 12 km
 (b) 10 km
 (c) 8 km
 (d) none of the above
13. A $4 \times 4 \times 4$ cubical box contains sixty-four identical small cubes that exactly fill the box. How many of these small cubes touch a side or the bottom of the box?

- (a) 48
 (b) 60
 (c) 52
 (d) none of the above

14. Find $1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \dots}}}$.

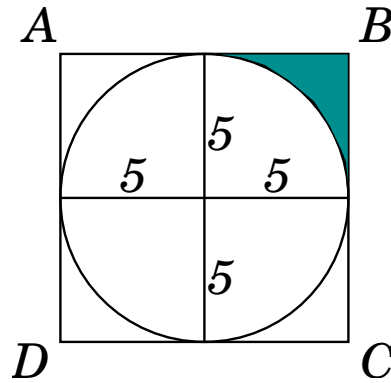
- (a) $\frac{1 + \sqrt{2}}{2}$
 (b) $\frac{1 - \sqrt{5}}{2}$
 (c) $\frac{1 + \sqrt{3}}{2}$
 (d) none of the above

15. It takes 1140 digits to number all pages of a book. How many pages are there in the book?

- (a) 416
 (b) 317
 (c) 450
 (d) none of the above

16. Find the value of the sum
 $2^{-1} + 2^{-2} + 2^{-3} + \dots + 2^{-9} + 2^{-10}$.
- (a) $\frac{1000}{1024}$
- (b) $\frac{1020}{1024}$
- (c) $\frac{1000}{4441}$
- (d) none of the above
17. Consider the smallest integer greater than 1000 that is divisible by 5 and 13 but not by 4. What other number is it divisible by?
- (a) 16
- (b) 17
- (c) 18
- (d) none of the above
18. What is the minimum number of students in a class that guarantees that at least 6 students are of the same sex?
- (a) 6
- (b) 11
- (c) 12
- (d) none of the above
19. If the diagonal of a face of a cube is 4 inches, find the diagonal of the cube.
- (a) $6\sqrt{2}$
- (b) $3\sqrt{2}$
- (c) $2\sqrt{6}$
- (d) none of the above
20. In a certain school, the ratio of male to female students is 3 to 2. If 15% of the male students and 10% of the female students are graduating, what percent of the total student body is graduating?
- (a) 2.5%
- (b) 4%
- (c) 13%
- (d) none of the above

21. Given the square $ABCD$, find the area of the shaded region.



- (a) 20
- (b) 25
- (c) 40
- (d) none of the above

22. Solve for x : $4^x - 4^{x-1} = 12$

- (a) 2
- (b) 3
- (c) 9
- (d) none of the above

23. What percent of $\sqrt{5}$ is 6?

- (a) $60\sqrt{5}$
- (b) $120\sqrt{5}$
- (c) $20\sqrt{5}$
- (d) none of the above

24. In how many distinguishable ways can the letters of the word CINCINNATI be arranged?

- (a) 45,850
- (b) 36,520
- (c) 50,400
- (d) none of the above

25. The owner of a bicycle shop took inventory of his bicycles and tricycles. He counted 153 wheels and 136 pedals. How many bicycles did he have?

- (a) 68
- (b) 17
- (c) 51
- (d) none of the above

26. If y is inversely proportional to the square of x and $y = 5$ when $x = 5$, what is y when $x = \frac{1}{10}$?

- (a) 2150
- (b) 250
- (c) 1250
- (d) none of the above

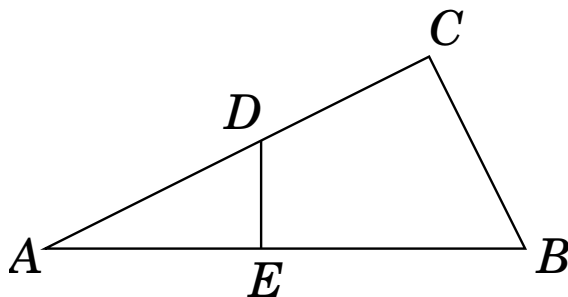
27. The sum of 101 consecutive integers is equal to 101. What is the largest integer in the sequence?

- (a) 51
- (b) 60
- (c) 47
- (d) none of the above

28. John counted the number of subsets that a set X has, and Mary counted the number of subsets that Y has. If John counted 96 more subsets than Mary, how many elements does X have?

- (a) 5
- (b) 9
- (c) 3
- (d) none of the above

29. In the figure below, AB is perpendicular to DE and AC is perpendicular to CB . $AE = 6$, $EB = 7$, and $BC = 5$. What is the area of the quadrilateral $EBCD$?

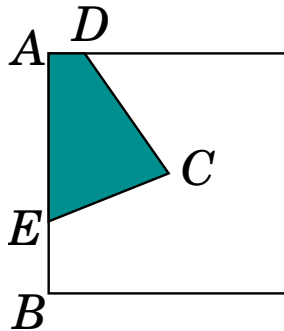


- (a) 12
- (b) 45
- (c) 22.5
- (d) none of the above

30. An isosceles triangle with equal sides of length 5 cm and base of length 6 cm is inscribed in a circle. What is the radius of the circle?

- (a) 12.5
- (b) 5
- (c) 2.5
- (d) none of the above

31. A culture of bacteria doubles in size twice every day. A dish with 1 thousand bacteria is full after 15 days. How long will it take for a dish with 2 thousand bacteria to fill?
- (a) 10.5 days
 (b) 6.5 days
 (c) 14.5 days
 (d) none of the above
32. The number $2^7 \times 3^4 \times 5 \times 7^2 \times 11^3$ is divisible by how many perfect squares?
- (a) 36
 (b) 48
 (c) 60
 (d) none of the above
33. During a basketball game, Jenny made 60 percent of her free throws and Ellen made 75 percent of her free throws. If together they made twelve of eighteen free throws, how many free throws did Jenny attempt?
- (a) 12
 (b) 10
 (c) 15
 (d) none of the above
34. In the figure below C is the center of a $100 \text{ cm} \times 100 \text{ cm}$ square. AD is 17 cm. Find the distance of EB so that $ADCE$ is $1/5$ of the area of the square.



- (a) 37
 (b) 41
 (c) 32
 (d) none of the above
35. In a group of 30 people there are 20 women and 10 men. The average height of the women is $66''$ and the average height of the men is $69''$. What is the average height of 30 people?
- (a) $67''$
 (b) $68''$
 (c) $67\frac{1}{2}''$
 (d) none of the above

36. Nine woodchucks can chuck eight pieces of wood in three hours. How much wood can a woodchuck chuck in one hour?
- (a) $8/3$ pieces
 - (b) $8/9$ pieces
 - (c) $8/27$ pieces
 - (d) none of the above
37. A moth is sitting in the lower-left-front corner of a storage shed having dimensions $8 \text{ ft} \times 12 \text{ ft} \times 9 \text{ ft}$. What is the length of the shortest path that the moth can fly to the upper-right-back corner of the shed?
- (a) 14.40 ft
 - (b) 17.00 ft
 - (c) 12.04 ft
 - (d) none of the above
38. During a special sale, a car was sold for \$4200. This price was \$200 more than $2/3$ of the normal price. By what percent of the normal price had the car's price been reduced?
- (a) 40 percent
 - (b) 70 percent
 - (c) 30 percent
 - (d) none of the above
39. Today my daughter is $1/3$ of my age. Five years ago she was $1/4$ of my age then. How old is my daughter now?
- (a) 25
 - (b) 12
 - (c) 20
 - (d) none of the above
40. When the expression $15^6 \times 28^5 \times 55^7$ is evaluated it ends with a string of consecutive zeros. How many zeros are there in the string?
- (a) 18
 - (b) 10
 - (c) 4
 - (d) none of the above

MAD HATTER MARATHON B

PART II ANSWERS

- | | | | |
|-----|---|-----|---|
| 1. | b | 21. | d |
| 2. | c | 22. | a |
| 3. | a | 23. | b |
| 4. | a | 24. | c |
| 5. | d | 25. | c |
| 6. | c | 26. | d |
| 7. | c | 27. | a |
| 8. | b | 28. | d |
| 9. | b | 29. | c |
| 10. | b | 30. | d |
| 11. | b | 31. | c |
| 12. | b | 32. | b |
| 13. | c | 33. | b |
| 14. | d | 34. | a |
| 15. | a | 35. | a |
| 16. | d | 36. | c |
| 17. | b | 37. | b |
| 18. | b | 38. | c |
| 19. | c | 39. | d |
| 20. | c | 40. | b |