Math 142 **Summer 2000**

Practice Final

The actual Final exam will be shorter. No calculators will be allowed.

1. Evaluate the following integrals:

$$\int e^x \cos(2x)dx, \quad \int_0^1 x^2 e^x dx$$

2. Use tables to evaluate the following integrals:

$$\int \frac{dx}{\sqrt{16-25x^2}}, \quad \int \sin^3(2x)dx$$

3. Evaluate the following integrals:

$$\int \frac{x^2dx}{(x-1)(x-2)}, \quad \int \frac{(x+2)dx}{x^2(x+1)}, \quad \int \frac{dx}{x(x-1)(x-2)}$$

4. Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

$$\int_2^\infty e^{-x}dx, \quad \int_1^\infty \frac{1+|\sin x|}{x}dx, \quad \int_1^2 \frac{dx}{\sqrt{2-x}}$$

5. (a) Prove the following reduction formula:

$$\int \cos^n x dx = \frac{1}{n}\cos^{n-1}x \sin x + \frac{n-1}{n}\int \cos^{n-2}x dx$$

(b) Use this reduction formula to evaluate the integral:

$$\int \cos^4 x dx$$

(c) Use part (b) to evaluate the definite integral:

$$\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \cos^4 x dx$$

6. Find the areas enclosed by the following curves:

   (a) $y = x^2 - 4$ and $y = -3x^2$.

   (b) $x + y^2 = 2$ and $x + y = 0$.

7. (a) Find the volume of the solid obtained by rotating the region bounded by $y = x^2$, $y = 4$, $x = 0$, and $x = 2$ about the y-axis.

   (b) Find the volume of a pyramid with height h and rectangular base with dimensions b and $2b$.

8. A cable that weights 2 lb/ft is used to lift 800 lb of coal up a mineshaft 500 ft deep. Find the work done.

9. Graph the curve and find its exact lenght:

$$x = e^{t/4} \cos t, \quad y = e^{t/4} \sin t, \quad 0 \leq t \leq 2\pi.$$

10. (a) Find the average value of $f(x) = 4x - x^2$ on $[0, 3]$.

    (b) Sketch the graph of $f(x)$.

11. Find the area of the surface obtained by rotating the curve

$$y = \sin x, \qquad 0 \leq x \leq \pi$$

    about the x-axis.

12. Find the centroid of the triangle with vervices (-1,0), (1,0), and (0,1).

Answers

1.
$$\frac{2}{5}e^x \sin{(2x)} + \frac{1}{5}e^x \cos{(2x)} + C, \qquad e-2$$

2.
$$\frac{1}{5}\arcsin{(\frac{5}{4}x)} + C, \quad \frac{1}{6}\cos^3{(2x)} - \frac{1}{2}\cos{(2x)} + C$$

3.
$$x - \ln|x-1| + 4\ln|x-2| + C, \quad -\ln|x| - \frac{2}{x} + \ln|x+1| + C,$$

$$\frac{1}{2}\ln|x| - \ln|x-1| + \frac{1}{2}\ln|x-2| + C$$

4. e^{-2}, diverges, 2.

5. (a) Hint: integrate by parts.
   (b) $\frac{1}{4}\cos^3 x\sin x + \frac{3}{8}\cos x \sin x + \frac{3}{8}x + c$
   (c) $\frac{8\pi - 9\sqrt{3}}{32}$

6. (a) $\frac{16}{3}$
   (b) 4.5

7. (a) 8π
   (b) $\frac{2hb^2}{3}$

8. 650,000 ft-lb

9. $\sqrt{17}(e^{\frac{\pi}{2}} - 1)$

10. (a) 3

11. $2\pi(\sqrt{2} + \ln(\sqrt{2}+1))$

12. $(0, \frac{1}{3})$

Supplementary problems

Unless you have LOTS of free time, you probably won't have a chance to attempt all the problems in this list. Make sure you try some from each section which seems difficult for you. Only odd-numbered problems appear in the list below, since the answers are in the back of the book. However, if you want even more practice, you can try similar even-numbered problems, too.

- **Chapter 4 Review Exercises** 5, 9, 17, 23, 55, 67
- **Chapter 5 Review Exercises** 11, 27, 35, 49, 51
- **6.1** 3, 5, 21, 45
- **6.2** 5, 15, 31, 47
- **6.4** 3, 11, 17
- **6.5** 1, 11, 17
- **Chapter 6 Review Exercises** 1, 9, 21, 31
- **7.1** 9, 21, 47, 51
- **7.2** 11, 19, 53, 59
- **7.3** 3, 17, 33, 41
- **7.4** 3, 15, 43, 63
- **7.5** 9, 19, 35, 75
- **7.6** 5, 11, 25, 33
- **7.8** 9, 17, 29, 41
- **Chapter 7 Review Exercises** 9, 23, 39, 55, 73, 79
- **8.1** 3, 11, 21, 35
- **8.2** 3, 7, 19, 31
- **8.3** 21, 37

- **Chapter 8 Review Exercises** 1, 7, 11, 13, 15

As always, please feel free to ask for help if you need it.

GOOD LUCK !