EXERCISES FOR CHAPTER 7

Section 7.2: Revisiting Quantified Statements

7.1. (a) Express the following quantified statement in symbols:
For every odd integer n, the integer $3n + 1$ is even.
(b) Prove that the statement in (a) is true.

7.2. (a) Express the following quantified statement in symbols:
There exists a positive even integer n such that $3n + 2^{n-2}$ is odd.
(b) Prove that the statement in (a) is true.

7.3. (a) Express the following quantified statement in symbols:
For every positive integer n, the integer n^{n-1} is even.
(b) Show that the statement in (a) is false.

7.4. (a) Express the following quantified statement in symbols:
There exists an integer n such that $3n^2 - 5n + 1$ is an even integer.
(b) Show that the statement in (a) is false.

7.5. (a) Express the following quantified statement in symbols:
For every integer $n \geq 2$, there exists an integer m such that $n < m < 2n$.
(b) Prove that the statement in (a) is true.

7.6. (a) Express the following quantified statement in symbols:
There exists an integer n such that $m(n - 3) < 1$ for every integer m.
(b) Prove that the statement in (a) is true.

7.7. (a) Express the following quantified statement in symbols:
For every integer n, there exists an integer m such that $(n - 2)(m - 2) > 0$.
(b) Express in symbols the negation of the statement in (a).
(c) Show that the statement in (a) is false.

7.8. (a) Express the following quantified statement in symbols:
There exists a positive integer n such that $-nm < 0$ for every integer m.
(b) Express in symbols the negation of the statement in (a).
(c) Show that the statement in (a) is false.

7.9. (a) Express the following quantified statement in symbols:
For every positive integer a, there exists an integer b with $|b| < a$ such that $|bx| < a$ for every real number x.
(b) Prove that the statement in (a) is true.

7.10. (a) Express the following quantified statement in symbols:
For every real number x, there exist integers a and b such that $a \leq x \leq b$ and $b - a = 1$.
(b) Prove that the statement in (a) is true.

7.11. (a) Express the following quantified statement in symbols:
There exists an integer n such that for two real numbers x and y, $x^2 + y^2 \geq n$.
(b) Prove that the statement in (a) is true.

7.12. (a) Express the following quantified statement in symbols:
For every even integer a and odd integer b, there exists a rational number c such that either $a < c < b$ or $b < c < a$.
(b) Prove that the statement in (a) is true.

7.13. (a) Express the following quantified statement in symbols:
There exist two integers a and b such that for every positive integer n, $a < \frac{1}{n} < b$.
(b) Prove that the statement in (a) is true.

7.14. (a) Express the following quantified statement in symbols:
There exist odd integers a, b, and c such that $a + b + c = 1$.
(b) Prove that the statement in (a) is true.

7.15. (a) Express the following quantified statement in symbols:
For every three odd integers a, b, and c, their product abc is odd.
(b) Prove that the statement in (a) is true.