MATH 110 **Homework 11**

1. (20%) Express the following compound statement in terms of just conjunction and negation:

$$(P \vee Q) \to R.$$

2. (20%) On the island of Knights and Knaves, you meet three friends, Karl, Lars, and Mark. You know that one of them is a knight, one is a knave, and one is a tourist. They make the following statements.
Karl: "Mark is a knave."
Lars: "Karl is a knight."
Mark: "I am a tourist."
Is it possible to determine from this information who is what?

3. (20%) Let $A = \emptyset$, $B = \{1\}$, $C = \{\emptyset, 1, 2\}$. List all elements of the following sets.
   (a) $A \cup C$
   (b) $C \cap \overline{B}$
   (c) $\mathcal{P}(B)$
   (d) $\mathcal{P}(B) \times C$

4. (40%) For each of the following statements, determine its truth value. Provide a proof.
   (a) $\exists x, y \in \mathbb{N}\ x^3 + y^3 = 7$
   (b) $\exists x, y \in \mathbb{Z}\ x^3 + y^3 = 7$
   (c) $\forall x \in \mathbb{R}\ \exists y \in \mathbb{R}\ x^3 + y^3 = 7$
   (d) $\exists y \in \mathbb{R}\ \forall x \in \mathbb{R}\ x^3 + y^3 = 7$
   (e) $\forall x \in \mathbb{Z}\ \exists y \in \mathbb{Z}\ x^3 + y^3 = 7$
   (f) $\exists! x \in \mathbb{R}\ \exists! y \in \mathbb{R}\ x^3 + y^3 = 7$
   (g) $\forall x \in \mathbb{Z}\ \exists y \in \mathbb{Z}\ xy = 7$
   (h) $\exists x \exists y \in \mathbb{Z}\ \forall z \in \mathbb{Z}\ x + yz = 7$