Homework 10 - Solutions

7.6. R is not reflexive because $(a, a) \notin R$.

R is not symmetric because $(a, b) \in R$ but $(b, a) \notin R$.

R is transitive because whenever $(x, y) \in R$ and $(y, z) \in R$, then $(x, z) \in R$ (this is true vacuously because there is only one pair in R).

7.8. R is reflexive because $|a - a| = 0 \leq 2$, thus $(a, a) \in R$ for any $a \in \mathbb{Z}$.

R is symmetric because if $(a, b) \in R$, then $|a - b| \leq 2$, then $|b - a| = |a - b| \leq 2$, thus $(b, a) \in R$.

R is not transitive because e.g. $|1 - 3| \leq 2$ and $|3 - 5| \leq 2$, but $|1 - 5| \not\leq 2$, thus $(1, 3) \in R$ and $(3, 5) \in R$, but $(1, 5) \notin R$.

7.10. R is not reflexive because $A \neq \emptyset$, therefore there exists an element $x \in A$. However, $R = \emptyset$, so $(x, x) \notin R$.

R is symmetric and transitive (these properties hold vacuously since there are no pairs in R).

7.18. Since R is an equivalence relation, it is symmetric and transitive. Since $a\ R\ b$ and R is symmetric, $b\ R\ a$. Since $c\ R\ d$ and R is symmetric, $d\ R\ c$. Since $b\ R\ a$, $a\ R\ d$, and R is transitive, $b\ R\ d$. Since $b\ R\ d$, $d\ R\ c$, and R is transitive, $b\ R\ c$.

7.22. The statement is false. R is not an equivalence relation because it is not transitive: e.g. if $a = 2$, $b = 6$, $c = 3$, then $a|b$ and $c|b$, but $a \not| c$ and $c \not| a$, so $(a, b) \in R$ and $(b, c) \in R$, but $(a, c) \notin R$.

7.26. The statement is false. For example, $R_1 = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3)\}$ and $R_1 = \{(1, 1), (1, 3), (2, 2), (3, 1), (3, 3)\}$ are equivalence relations on $A = \{1, 2, 3\}$, but $R_3 = R_1 \cup R_2 = \{(1, 1), (1, 2), (1, 3)(2, 1), (2, 2), (3, 1), (3, 3)\}$ is not an equivalence relation (it is not transitive: $(2, 1), (1, 3) \in R_3$, but $(2, 3) \notin R_3$).