MATH 111 **Fall 2007**

Homework 14 - Solutions

9.14. Proof by Mathamatical Induction.

Basis step: if $n = 4$, then $4! > 2^4$ is true.

Inductive step: assume that $k! > 2^k$ holds for some $k \in \mathbb{Z}$, $k \geq 4$. We will prove that $(k+1)! > 2^{k+1}$.

Since $k + 1 > 2$, $(k+1)! = k!(k+1) > 2^k(k+1) > 2^k \cdot 2 = 2^{k+1}$.

9.19. Proof by Mathamatical Induction.

Basis step: if $n = 0$, then $4|(5^0 - 1)$ is true.

Inductive step: assume that $4|(5^k - 1)$ for some $k \in \mathbb{Z}$, $k \geq 0$. We will prove that $4|(5^{k+1} - 1)$.

Since $4|(5^k - 1)$, $5^k - 1 = 4x$ for some $x \in \mathbb{Z}$. Then $5^{k+1} - 1 = 5^k \cdot 5 - 1 = 5^k \cdot 4 + 5^k - 1 = 5^k \cdot 4 + 4x = 4(5^k + x)$. Since $5^k + x \in \mathbb{Z}$, $4|(5^{k+1} - 1)$.

9.32. The first few terms of the sequence are: $x_1 = 1$, $x_2 = 2$, $x_3 = 4$, $x_4 = 8$, $x_5 = 16$.

Conjecture: $x_n = 2^{n-1}$.

Proof by Mathematical Induction.

Basis step: if $n = 1$, then $x_1 = 2^0$ is true.

Inductive step: assume that $x_k = 2^{k-1}$ holds for some $k \in \mathbb{N}$. We will prove that $x_{k+1} = 2^k$.

Observe that $x_{k+1} = 2x_k = 2 \cdot 2^{k-1} = 2^k$.

9.40. The start is good, but after the phrase "observe that", the authors start with the identity that has to be proved, and derive a true statement ($(k+1)^2 = (k+1)^2$). The order is wrong here. We should either start with a true statement and derive the one we have to prove, or say "follows from"/"since"/"because"/ between the last four lines.