MATH 111 **Fall 2007**

Practice Test 3

Note: the actual test will consist of five or six questions.

1. This test is primarily on chapters 7-9, however, knowledge of previously covered material may be required. Review all terms, notations, and types of proofs in chapters 0–9.
2. Let $A = \{1, 2, 3, 4\}$ and $B = \{a, b, c\}$. Which of the following are relations from A to B or relations from B to A? Which of them are functions?
   (a) $\{(a, 1), (b, 2), (c, 3)\}$
   (b) $\{(1, b), (1, c), (3, a), (4, b)\}$
3. Determine which of the following relations are reflexive; symmetric; transitive. Which of them are equivalence relations? For those that are, describe the distinct equivalence classes.
   (a) Relation R on set $\mathbb{Z}$ defined by $(a, b) \in R$ iff $a + b = 0$.
   (b) Relation R on set $\mathbb{R}$ defined by $(a, b) \in R$ iff $\dfrac{a}{b} \in \mathbb{Q}$.
   (c) Relation R on set $\mathbb{R}$ defined by $(a, b) \in R$ iff $ab > 0$.
   (d) Relation R on set $\mathbb{Z}$ defined by $(a, b) \in R$ iff $a \equiv b \pmod 3$.
   (e) Relation R on set $\mathbb{Q}$ defined by $(a, b) \in R$ iff $a > b$.
4. Determine which of the following functions are one-to-one; onto; bijective.
   (a) $f : \mathbb{Z} \to \mathbb{Z}$ defined by $f(n) = 5n^2 + 2$.
   (b) $f : \mathbb{N} \to \mathbb{R}$ defined by $f(n) = \frac{1}{n}$.
   (c) $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \begin{cases} \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$.
   (d) $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^3 - x$.
5. Prove or disprove the following statements.
   (a) Let $f : A \to B$ and $g : B \to C$ be two functions. If g is onto, then $g \circ f$ is onto.
   (b) Let $f : A \to B$ and $g : B \to C$ be two functions. If both g and $g \circ f$ are one-to-one, then f is one-to-one.
   (c) Let $f : A \to B$ and $g : B \to C$ be two functions. If both f and $g \circ f$ are one-to-one, then g is one-to-one.
6. Use Mathematical Induction to prove the following statements.
   (a) Let $n \in \mathbb{N}$. Then $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \ldots + n(n + 1) = \dfrac{n(n + 1)(n + 2)}{3}$.
   (b) Let $f(x) = xe^{-x}$. Then $f^{(n)}(x) = (-1)^n e^{-x}(x - n)$ for every positive integer n.
   (c) Let $n \in \mathbb{N}$. Then $5|(n^5 - n)$.