

MATH 111

Test 2

November 5, 2007

Name: _____

- No books, notes, or calculators are allowed.
- Please show all your work.

1. (10 points) Let $a \in \mathbb{Z}$. Prove that if $4|a^2$, then $2|a$.

2. (10 points) Prove that $\sqrt[3]{2}$ is an irrational number.

3. (10 points) Prove or disprove.

The equation $x^3 + 5x + 2 = 0$ has a real solution.

4. (10 points) Prove or disprove.

Let A and B be sets. Then $(A - B) \cap (A \cup B) = A$.

5. (10 points) Prove or disprove.

For any integer a there exist an integer b such that $b < a$ and $a \equiv b \pmod{2}$.

6. (**For extra credit**, 8 points) Prove or disprove.

The number $\frac{\sqrt{2} - 1}{\sqrt{2} + 1}$ is irrational.