

# MATH 111

## Test 2

November 6, 2007

Name: \_\_\_\_\_

- No books, notes, or calculators are allowed.
- Please show all your work.

1. (10 points) Let  $a \in \mathbb{Z}$ . Prove that if  $3|a^2$ , then  $3|a$ .

2. (10 points) Prove that for any integer  $a$ , there exists an integer  $b$  such that  $b > a$  and  $a \equiv b \pmod{5}$ .

3. (10 points) Prove or disprove.

The equation  $x^3 + 5x + 2 = 0$  has an integer solution.

4. (10 points) Prove or disprove.

Let  $A$  and  $B$  be sets. Then  $(A - B) \cup (A \cap B) = A$ .

5. (10 points) Prove or disprove.

There exists a largest rational number, i.e. a rational number  $a$  such that for any rational number  $b$ ,  $a \geq b$ .

6. (**For extra credit**, 8 points) Prove or disprove.

For any rational numbers  $a$  and  $b$  such that  $a < b$ , there exists an irrational number  $x$  such that  $a < x < b$ .