MATH 111

Test 1

February 27, 2006

Name:____

- No books, notes, or calculators are allowed.
- Please show all your work.

1. (12 points) Let $\mathbb{R}$ be the universal set, and let $A = [0, 3)$ and $B = (-\infty, 2)$.

   (a) Determine and write in the interval notation the following sets:

      i. $A \cup B$

      ii. $\overline{B}$

      iii. $\overline{A} \cap B$

   (b) How many elements does A have?

2. (6 points) Let P and Q be propositions. Are compound propositions $P \Rightarrow Q$ and $P \vee \neg Q$ logically equivalent? If so, prove it. If not, provide an example of P and Q for which one of these compound propositions is true and the other one is false.

3. (9 points) Let $x \in \mathbb{R}$, and let $P(x, y)$ denote "$x \geq y + 2$". Determine the truth values of the following propositions. (Explain your answers!)

   (a) $\forall x \exists y P(x, y)$

   (b) $\exists y \forall x P(x, y)$

   (c) $\exists! x P(x, 1)$

4. (8 points) Which of the following implications can be proved using a trivial proof? Prove it (use a trivial proof).

- Let $x \in \mathbb{R}$. If $x^2 < -25$, then $x < -5$.
- Let $n \in \mathbb{Z}$. If $8 < n \leq 39$, then $6n + 4$ is even.
- Let $n \in \mathbb{Z}$. If n is odd, then $4n$ and $5n$ are of opposite parity.

5. (5 points) Let $n \in \mathbb{N}$. Prove that $4n^2 - 6n - 3$ is an odd integer.

6. (10 points) Let $n \in \mathbb{N}$. Prove that $5n + 3$ is odd if and only if n is even.

7. (**For extra credit**, 8 points)

(a) Give an example of a family of sets A_n (where $n \in \mathbb{N}$) such that $\cup_{n\in\mathbb{N}} A_n = \mathbb{R}$ and $\cap_{n\in\mathbb{N}} A_n = \mathbb{Z}$.

(b) What are $\cup_{n=3}^{5} A_n$ and $\cap_{n=3}^{5} A_n$ for your sets?