MATH 111 **Spring 2006**

Homework 3 (due Wed, Feb 15)
Revised: 2/6/06

1. Let $P(x)$ denote "$x = -2$" and let $Q(x)$ denote "$x^2 = 4$" (as in exercise 2.16 in the book). Determine (and explain!) the truth values of the following statements:
   (a) $\exists x \neg P(x)$
   (b) $\forall x (P(x) \vee Q(x))$
   (c) $\exists x (P(x) \wedge Q(x))$
   (d) $\forall x (P(x) \Rightarrow Q(x))$
   (e) $\exists x (Q(x) \Rightarrow P(x))$
   (f) $\forall x (P(x) \Leftrightarrow Q(x))$

2. Let $F(x, y)$ be statement "x can fool y", where the domain for both variables is the set of all people in the world. Use quantifiers to express each of the following statements:
   (a) Mike can fool everybody.
   (b) Everybody can fool somebody.
   (c) No one can fool both Fred and Jerry.

3. Let $Q(x, y)$ denote "$x + y = 0$". What are the truth values of the statements $\exists y \forall x Q(x, y)$ and $\forall x \exists y Q(x, y)$? Explain your answers!

4. Rewrite each of the following statements so that negations appear only within predicates:
   (a) $\neg \forall y \exists x P(x, y)$
   (b) $\neg (\exists x \exists y \neg P(x, y) \wedge \forall x \forall y Q(x, y))$

5. Let $x \in \mathbb{R}$. Prove that if $-2 < x < 4$, then $x^2 + 2x + 4 \geq 3$.

6. Let $n \in \mathbb{Z}$. Prove that if $n^2 - 2n + 5 \leq 3$, then n is even.

7. Prove that if n is an integer, then $2n^2 - 8n + 10$ is an even integer.

8. (Problem 3.2 on page 64.) Prove that if x is an even integer, then $5x - 3$ is an odd integer.