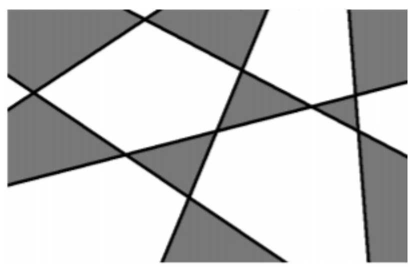
Math 111 **Fun problems on Mathematical Induction**

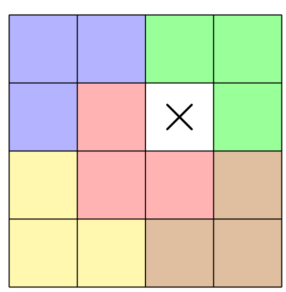
(for extra credit, due April 4, 2024)

Use Mathematical Induction (regular or strong) to prove the following statements. These problems may be done individually or in a small group. Your solutions may be typed or hand-written (or a combination of both). You will get 2 points of extra credit for each correct solution.

1. Prove that for any $n \in \mathbb{N}$ lines in the plane, it is possible to color the regions formed by these lines with two colors so that no regions with a common boundary line are colored the same way. (Regions that share only one boundary point may be colored the same way.) The picture below shows an example of such a coloring for six lines.



2. Prove that for any natural number n, if any unit square is removed from a $2^n \times 2^n$ board, then the remaining part can be completely covered by L-shaped tiles consisting of three squares each (without overlap or any tiles sticking out). For example, if $n = 2$ and the unit square in the second row and third column is removed, the picture below shows how the rest of the 4×4 board can be covered by such tiles.



3. It is easy to cut a square into four smaller squares. However, it is impossible to cut a square into two, three, or five smaller squares (this is not very easy to prove, and you are not asked to do that). Prove that for any natural number $n \geq 6$, it is possible to cut a square into n smaller squares (not necessarily all of the same size).