MATH 111

Practice Test 2

Note: the actual test will consist of five or six questions.

1. This test is primarily on chapters 4-6, however, knowledge of previously covered material may be required. Review all terms, notations, and types of proofs in chapters 0–6.
2. Prove the following statements. (Some statements have alternate wording/notation. You should be comfortable with either wording/notation.) Indicate what type of proof (e.g. direct, by contrapositive, or by contradiction) you used.
   (a) If n is an integer such that $5 \mid (n-1)$, then $5 \mid (n^3+n-2)$.
   (Let $n \in \mathbb{Z}$. If $n-1 \equiv 0 \pmod 5$, then $n^2+n-2 \equiv 0 \pmod 5$.)
   (b) The number $\log_3 2$ is irrational.
   (c) Let $n \in \mathbb{Z}$. If $7n^2+4$ is even, then n is even.
   (d) Let $x \in \mathbb{R}$. If $2x > x^2+x^3$, then $x < 1$.
   (e) Let $m, n \in \mathbb{Z}$. Then $3 \mid (mn)$ if and only if $3 \mid m$ or $3 \mid n$.
   (Let $m, n \in \mathbb{Z}$. Then $mn \equiv 0 \pmod 3$ if and only if $m \equiv 0 \pmod 3$ or $n \equiv 0 \pmod 3$.)
   (f) The product of a nonzero rational number and an irrational number is irrational.
   (g) Let $a, b, c \in \mathbb{Z}$. If $a \nmid (bc)$, then $a \nmid b$ and $a \nmid c$.
   (h) Let A and B be sets. Then $A \cap B = \emptyset$ if and only if $(A \times B) \cap (B \times A) = \emptyset$.
3. Use Mathematical Induction to prove the following statements.
   (a) Let $n \in \mathbb{N}$. Then $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + n(n+1) = \dfrac{n(n+1)(n+2)}{3}$.
   (b) Let $f(x) = xe^{-x}$. Then $f^{(n)}(x) = (-1)^n e^{-x}(x-n)$ for every positive integer n.
   (c) Let $n \in \mathbb{N}$. Then $5 \mid (n^5 - n)$.
   (Let $n \in \mathbb{N}$. Then $n^5 \equiv n \pmod 5$.)