

Practice Test 3

Note: the actual test will consist of five or six questions.

1. This test is primarily on chapters 8–10, however, knowledge of previously covered material may be required. Review all terms, notations, and types of proofs in chapters 0–10.
2. Prove or disprove the following statements.
 - (a) There exists a nonzero integer a such that for every real number b , $b^2 \geq a$.
 - (b) There exists an integer a such that $a^3 + 2a + 3 = 100$.
 - (c) For any integer a there exists an integer b such that $b^2 = a$.
 - (d) The sum of any two positive irrational numbers is irrational.
 - (e) Any irrational number is the sum of an irrational number and a positive rational number.
 - (f) For any sets A and B there exists a set C such that $A \cup C = B \cup C$.
 - (g) Let A, B, C , and D be sets such that $A \subset C$ and $B \subset D$. If $A \cap B = \emptyset$, then $C \cap D = \emptyset$.
 - (h) Let A, B, C , and D be sets such that $A \subset C$ and $B \subset D$. If $C \cap D = \emptyset$, then $A \cap B = \emptyset$.
3. Let $A = \{1, 2, 3, 4\}$ and $B = \{a, b, c\}$. Which of the following are relations from A to B or relations from B to A ? Which of them are functions?
 - (a) $\{(a, 1), (b, 2), (c, 3)\}$
 - (b) $\{(1, b), (1, c), (3, a), (4, b)\}$
4. Determine which of the following relations are reflexive; symmetric; transitive. Which of them are equivalence relations? For those that are, describe the distinct equivalence classes.
 - (a) Relation R on set \mathbb{Z} defined by $(a, b) \in R$ iff $a + b = 0$.
 - (b) Relation R on set \mathbb{R} defined by $(a, b) \in R$ iff $\frac{a}{b} \in \mathbb{Q}$.
 - (c) Relation R on set \mathbb{R} defined by $(a, b) \in R$ iff $ab > 0$.
 - (d) Relation R on set \mathbb{Z} defined by $(a, b) \in R$ iff $a \equiv b \pmod{3}$.
 - (e) Relation R on set \mathbb{Q} defined by $(a, b) \in R$ iff $a > b$.
5. Determine which of the following functions are one-to-one; onto; bijective.
 - (a) $f : \mathbb{Z} \rightarrow \mathbb{Z}$ defined by $f(n) = 5n^2 + 2$.
 - (b) $f : \mathbb{N} \rightarrow \mathbb{R}$ defined by $f(n) = \frac{1}{n}$.
 - (c) $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = \begin{cases} \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$.
 - (d) $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = x^3 - x$.