

## Math 114

### Optional Problems on Pigeonhole Principle

1. Suppose that five lattice points (i.e. points with integer coordinates) are given in the plane. Prove that we can choose two of these points so that the line segment joining these two points passes through another lattice point.
2. Six points are given inside a  $7 \times 8$  rectangle. Prove that among them there are at least two points with distance between each other less than or equal to 5.
3. Prove that in any set of  $n + 1$  positive integers not exceeding  $2n$  there must be two that are relatively prime.
4. (2pts) Every block of a  $5 \times 41$  board is colored in one of four colors. Prove that, no matter how the board is colored, there exists a rectangle consisting of more than one row and more than one column whose four corners have the same color.