

Math 114

Optional Problems on Pigeonhole Principle

1. Six points are given inside a 7×8 rectangle. Prove that among them there are at least two points with distance between each other less than or equal to 5.
2. (a) Prove that among 11 distinct natural numbers, there are two numbers $a < b$ such that the difference $b - a$ ends with 0 (i.e., has the units digit 0).
(b) Is the above statement true for the tens digit?
3. (2pts) We proved in class that every sequence of $n^2 + 1$ distinct real numbers has a monotone (i.e., increasing or decreasing) subsequence of length $n + 1$. Show that a subsequence of n^2 distinct real numbers may not have a monotone subsequence of length $n + 1$
 - (a) in the special case of $n = 3$, that is, give an example of a sequence of length 9 that does not have any monotone subsequence of length 4.
 - (b) in general, for any $n \geq 1$, that is, show how to construct a sequence of length n^2 that does not have any monotone subsequence of length $n + 1$.