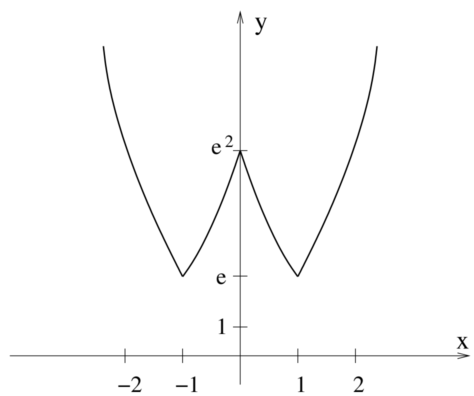
Math 145 **Fall 2003**

Homework 13 - Solutions

Calculus and Absolute values

1. **Sketch the graph of** $f(x) = e^{||x|-1|}$**.**



2. **Let** $f(x) = |1 + |x - 5| - x^3|$**. Find** $f'(2)$**.**

   When x is close to 2, $x - 5$ is negative, so $|x - 5| = -(x - 5) = 5 - x$.

   Then $f(x) = |1 + 5 - x - x^3| = |6 - x - x^3|$. Now, if x is close to 2, $6 - x - x^3$ is negative, thus

   $f(x) = -(6 - x - x^3) = x^3 + x - 6$. Then $f'(x) = 3x^2 + 1$, and $f'(2) = 13$.

3. **Evaluate the integral** $\int_{-2}^{2} ||x - 5| - 3 - x^2|dx$**.**

$$\int_{-2}^{2} ||x-5| - 3 - x^2|dx = \int_{-2}^{2} |-(x-5) - 3 - x^2|dx = \int_{-2}^{2} |2 - x - x^2|dx$$

$$= \int_{-2}^{2} |(2+x)(1-x)|dx = \int_{-2}^{1} |(2+x)(1-x)|dx + \int_{1}^{2} |(2+x)(1-x)|dx$$

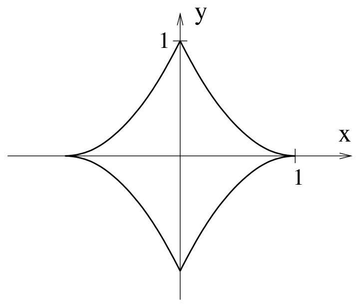
$$= \int_{-2}^{1} (2+x)(1-x)dx + \int_{1}^{2} (-(2+x)(1-x))dx$$

$$= \int_{-2}^{1} (2 - x - x^2)dx + \int_{1}^{2} (-2 + x + x^2)dx$$

$$= \left(2x - \frac{x^2}{2} - \frac{x^3}{3}\right)\Bigg|_{-2}^{1} + \left(-2x + \frac{x^2}{2} + \frac{x^3}{3}\right)\Bigg|_{1}^{2}$$

$$= \left(2 - \frac{1}{2} - \frac{1}{3}\right) - \left(-4 - 2 + \frac{8}{3}\right) + \left(-4 + 2 + \frac{8}{3}\right) - \left(-2 + \frac{1}{2} + \frac{1}{3}\right) = \frac{19}{3}$$

4. **Find the area bounded by the curve $|x| + \sqrt{|y|} = 1$.**



The region is symmetric about both axes, so we'll find the area of the region bounded by the given curve in the first quadrant, and multiply it by 4. In the first quadrant both x and y are positive, so we have $x + \sqrt{y} = 1$, or $\sqrt{y} = 1 - x$. Square both sides: $y = (1-x)^2$. Since the region lies between 0 and 1, its area is

$$\int_0^1 (1-x)^2 dx = \int_0^1 (1 - 2x + x^2) dx = \left(x - x^2 + \frac{x^3}{3}\right)\Big|_0^1 = \frac{1}{3}.$$

Thus the area of the whole region is $\frac{4}{3}$.

5. **Find a number m such that the area enclosed by $y = mx$ and $y = |x-1|$ is 2.**

First find the intersection points: $mx = |x-1|$.

If $x \geq 1$, then $|x-1| = x-1$, and we have $mx = x-1$. Then $x = -\frac{1}{m-1}$.

If $x < 1$, then $|x-1| = 1-x$, and we have $mx = 1-x$. Then $x = \frac{1}{m+1}$.

The area of the enclosed region is

$$\int_{\frac{1}{m+1}}^{1} (mx - (1-x))dx + \int_1^{-\frac{1}{m-1}} (mx - (x-1))dx$$

$$= \left((m+1)\frac{x^2}{2} - x\right)\Big|_{\frac{1}{m+1}}^{1} + \left((m-1)\frac{x^2}{2} + x\right)\Big|_1^{-\frac{1}{m-1}}$$

$$= \frac{m+1}{2} - 1 - \frac{1}{2(m+1)} + \frac{1}{m+1} + \frac{1}{2(m-1)} - \frac{1}{m-1} - \frac{m-1}{2} - 1$$

$$= \frac{1}{2(m+1)} - \frac{1}{2(m-1)} - 1 = \frac{-2m^2}{2(m+1)(m-1)}.$$

Now we set the area equal to 2: $\frac{-2m^2}{2(m+1)(m-1)} = 2.$

$-2m^2 = 4m^2 - 4$

$4 = 6m^2$

$m = \sqrt{\frac{2}{3}}$