Math 145 **Fall 2003**

Finding a pattern

Examples

Problem. Find the n-th derivative of $f(x) = 5^x$.

Solution. Find the first few derivatives (until you can see a pattern):
$f'(x) = \ln 5 \cdot 5^x$
$f''(x) = \ln 5 \cdot \ln 5 \cdot 5^x = (\ln 5)^2 \cdot 5^x$
$f'''(x) = (\ln 5)^2 \cdot \ln 5 \cdot 5^x = (\ln 5)^3 \cdot 5^x$
We notice that $f^{(n)}(x) = (\ln 5)^n \cdot 5^x$.
It is easy to prove this formula using Mathematical Induction.
The basis step is $f'(x) = \ln 5 \cdot 5^x$.
Inductive step: suppose $f^{(k)}(x) = (\ln 5)^k \cdot 5^x$ is true. Then
$f^{(k+1)}(x) = (f^{(k)}(x))' = ((\ln 5)^k \cdot 5^x)' = (\ln 5)^k \cdot \ln 5 \cdot 5^x = (\ln 5)^{k+1} \cdot 5^x$.

Note: When you notice a pattern, it is often needed to guess a formula (which you can then prove by Mathematical Induction or some other method). For the problems like the one below, since only a few terms are given, there may be several different formulas valid for these few terms. Try to find the simplest possible formula, but any correct formula (that is, any formula that works for the given terms) will be accepted.

Problem. Find a formula for the n-th term of the sequence: 1, 3, 6, 10, 15, 21, ...

Solution. Notice that the difference between the first and the second terms is 2, the difference between the second and the third terms is 3, and then the differences are 4, 5, 6, ... Thus
$a_1 = 1$,
$a_2 = 1 + 2$,
$a_3 = 1 + 2 + 3$,
$a_4 = 1 + 2 + 3 + 4$,
$a_5 = 1 + 2 + 3 + 4 + 5$,
$a_6 = 1 + 2 + 3 + 4 + 5 + 6$.
So it appears that $a_n = 1 + 2 + \ldots + n = \dfrac{n(n+1)}{2}$.