Math 145 **Fall 2005**

Practice Test 1

Answer the question (5 points):

- What does "a and b are relatively prime" mean?

and do any 3 of the following problems (15 points each):

1. Let $P(x, y)$ denote the proposition "$x \leq y$" where x and y are real numbers. Determine the truth values of
   (a) $\forall x P(-x, x)$,
   (b) $\exists x \exists y P(x, y)$,
   (c) $\forall x \exists y P(x, y)$,
   (d) $\exists x \forall y P(x, y)$,
   (e) $\forall x \forall y P(x, y)$.
2. Prove that for any integers n and m, if $nm + 2n + 2m$ is odd then both n and m are odd (you may only use the definitions of even and odd numbers; do not use any properties unless you prove them). Is your proof direct, by contrapositive, or by contradiction?
3. Prove that for any natural n,
$$1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \ldots + n(n+1) = \frac{n(n+1)(n+2)}{3}.$$
4. Kevin is paid every other week on Friday. Show that every year, in some month he is paid three times.

Extra credit (15 points):

- Let f be a one-to-one function from $X = \{1, 2, \ldots, n\}$ onto X. Let $f^k = \underbrace{f \circ f \circ \ldots \circ f}_{k \text{ times}}$ denote the k-fold composition of f with itself. Show that for some positive integer m, $f^m(x) = x$ for all $x \in X$.