

Practice Test 3

Do any 3 of the following problems:

1. Prove that a 6×6 board cannot be covered by L-tetrominoes.
2. A graph $K_{k,l,m}$ has $k+l+m$ vertices divided into three sets: k vertices in one set, l vertices in another set, and m vertices in the third set. Two vertices are connected if and only if they are in different sets. Prove that $K_{1,3,5}$ has a Hamilton path but not a Hamilton cycle.
3. Two players play the following game. Turns alternate. At each turn, a player removes 1, 2, or 4 coins from a pile that initially had 10 coins. The game ends when all coins have been removed. The player who cannot make a move loses. Find a winning strategy for one of the players.
4. Find an equation of the line with a negative slope and passing through the point $(1, 1)$ such that the triangle bounded by this line and the axes is divided by the parabola $y = x^2$ into two regions of equal area.

Extra credit:

- Is it possible for a chess knight to pass through all the squares of a 4×2019 board having visited each square exactly once, and return to the initial square?