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Problems and Solutions

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PROBLEMS AND SOLUTIONS

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This section contains problems intended to challenge students and teachers of college mathematics. We urge you to participate actively *both* by submitting solutions and by proposing problems that are new and interesting. To promote variety, the editors welcome problem proposals that span the entire undergraduate curriculum.

Proposed problems should be sent to **Greg Oman**, either by email (preferred) as a pdf, TEX, or Word attachment or by mail to the address provided above. Whenever possible, a proposed problem should be accompanied by a solution, appropriate references, and any other material that would be helpful to the editors. Proposers should submit problems only if the proposed problem is not under consideration by another journal.

Solutions to the problems in this issue should be sent to **Chip Curtis**, either by email as a pdf, TEX, or Word attachment (preferred) or by mail to the address provided above, no later than May 15, 2022. Sending both pdf and TEX files is ideal.

PROBLEMS

1211. *Proposed by Necdet Batir, Nevşehir Haci Bektaş Veli University, Nevşehir, Turkey.*

Evaluate the following limit, where below, $H_0 = 0$ and for $n > 0$, H_n denotes the nth harmonic number $\sum_{k=1}^{n} \frac{1}{k}$:

$$\lim_{n\to\infty}\left((H_n)^2 - \sum_{k=1}^{n} \frac{H_{n-k}}{k}\right).$$

1212. *Proposed by Paul Bracken, University of Texas, Edinburg, TX.*

Let n be an odd natural number and let $\theta \in \mathbb{R}$ be such that $\cos(n\theta) \neq 0$. Prove the following:

$$\sum_{k=0}^{n-1} \frac{\sin\theta}{\sin^2\theta - \cos^2(\frac{k\pi}{n})} = -\frac{n\sin(n\theta)}{\cos\theta\cos(n\theta)}, \text{ and} \tag{1}$$

$$\sum_{k=0}^{n-1} \frac{(-1)^{k+1}\cos(\frac{k\pi}{n})}{\sin^2\theta - \cos^2(\frac{k\pi}{n})} = \frac{n\sin(\frac{n\pi}{2})}{\cos\theta\cos(n\theta)}. \tag{2}$$

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1213. *Proposed by Rafael Jakimczuk, Universidad National de Lujá, Buenos Aires, Argentina.*

Let (a_n) be a sequence of positive integers, and for every positive integer n, define $P_n := (1 + \frac{1}{a_1 n})^{a_1} \cdot (1 + \frac{1}{a_2 n})^{a_2} \cdots (1 + \frac{1}{a_n n})^{a_n}$. Find $\lim_{n\to\infty} P_n$.

1214. *Proposed by Luis Moreno, SUNY Broome Community College, Binghampton, NY.*

The following sequence can be found in the text *Intermediate Analysis* by John Olmsted: $(1, 2, 2\frac{1}{2}, 3, 3\frac{1}{3}, 3\frac{2}{3}, 4, 4\frac{1}{4}, 4\frac{2}{4}, 4\frac{3}{4}, 5, \ldots)$. Now let n be a positive integer. Find a closed-form expression for a_n, the nth term of the above sequence.

1215. *Proposed by Greg Oman, University of Colorado at Colorado Springs, Colorado Springs, CO.*

Let R be a ring (assumed only to be associative but not to contain an identity unless stated). Recall that a *subring* of R is a nonempty subset of R closed under addition, negatives, and multiplication. Find all rings R with identity $1 \neq 0$ with the property that no proper, nontrivial subring of R has an identity (which need NOT be the identity of R).

SOLUTIONS

A continued fraction given by Fibonacci

1186. *Proposed by Gregory Dresden, Washington and Lee University, Lexington, VA and ZhenShu Luan (high school student), St. George's School, Vancouver, BC, Canada.*

Find a closed-form expression for the continued fraction $[1, 1, \ldots, 1, 3, 1, 1, \ldots, 1]$, which has n ones before, and after, the middle three.

Solution by Walther Janous, Ursulinengymnasium, Innsbruck, Austria.

In order to get the desired expression, we recall the following elegant way of evaluating the convergents of a continued fraction. [See, for instance, https://de.wikipedia.org/wiki/Kettenbruch, particularly the paragraph "matrixdarstellung."] We have to evaluate the product

$$\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n \cdot \begin{bmatrix} 3 & 1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n$$

Let F_n be the nth Fibonacci number. From the familiar representation

$$[1, 1, ..., 1] = \frac{F_{n+1}}{F_n},$$

(with n 1's), we get

$$\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n = \begin{bmatrix} F_{n+1} & F_n \\ F_n & F_{n-1} \end{bmatrix},$$

whence

$$\begin{bmatrix} 1 & \\ 1 & 0 \end{bmatrix}^n \cdot \begin{bmatrix} 3 & 1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n = \begin{bmatrix} F_{n+1} & F_n \\ F_n & F_{n-1} \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} F_{n+1} & F_n \\ F_n & F_{n-1} \end{bmatrix};$$

that is

$$\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n \cdot \begin{bmatrix} 3 & 1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n$$
$$= \begin{bmatrix} F_{n+1} \cdot (3F_{n+1} + 2F_n) & F_{n+1} \cdot F_{n-1} + F_n (3F_{n+1} + F_n) \\ F_{n+1} \cdot F_{n-1} + 3F_n F_{n+1} + F_n^2 & F_n (2F_{n-1} + 3F_n) \end{bmatrix}.$$

This leads to the desired closed-form expression of [1, ..., 1, 3, 1, ..., 1]:

$$\begin{aligned} \frac{F_{n+1}(3F_{n+1} + 2F_n)}{F_{n+1} \cdot F_{n-1} + 3F_n \cdot F_{n+1} + F_n^2} &= \frac{F_{n+1}(3F_{n+1} + 2F_n)}{F_{n+1}(F_{n+1} - F_n) + F_n(3F - n + 1 + F_n)} \\ &= \frac{F_{n+1}(3F_{n+1} + 2F_n)}{F_{n+1}^2 + 2F_{n+1} \cdot F_n + F_n^2} \\ &= \frac{F_{n+1}(3F_{n+1} + 2F_n)}{(F_{n+1} + F_n)^2} \\ &= \frac{F_{n+1}(3F_{n+1} + 2F_n)}{F_{n+2}^2} \\ &= \frac{F_{n+1}(F_{n+1} + 2F_{n+2})}{F_{n+2}^2}. \end{aligned}$$

This and

$$F_{n+1} + 2F_{n+2} = F_{n+3} + F_{n+2} = F_{n+4}$$

yield the closed-form result

$$\frac{F_{n+1}F_{n+4}}{F_{n+2}^2}.$$

Also solved by Brian Beasley, Presbyterian C.; Anthony Bevelacqua, U. of N. Dakota; Brian Bradie, Christopher Newport U.; James Brenneis, Penn State - Shenango; Hongwei Chen, Christopher Newport U.;Giuseppe Fera, Vicenza, Italy; Eugene Herman, Grinnell C.; Donald Hooley, Bluffton, OH; Joel Iiams, U. of N. Dakota; Harris Kwong, SUNY Fredonia; Seungheon Lee, Yonsei U.; Carl Libis, Columbia Southern U.; Graham Lord, Princeton, NJ; Ioana Mihaila, Cal Poly Pomona; Missouri State U. Problem Solving Group; Northwestern U. Math Problem Solving Group; Randy Schwartz, Schoolcraft C. (retired); Albert Stadler, Herrliberg, Switzerland; Paul Stockmeyer, C. of William and Mary; David Terr, Oceanside, CA; Enrique Treviño, Lakeforest C.; Michael Vowe, Therwil, Switzerland; and the proposer.

A limit of maxima

1187. *Proposed by Reza Farhadian, Lorestan University, Khorramabad, Iran.*

Let $\alpha > 1$ be a fixed real number, and consider the function $M\colon [1, \infty) \to \mathbb{N}$ defined by $M(x) = \max\{m \in \mathbb{N} \colon m! \le \alpha^x\}$. Prove the following:

$$\lim_{n\to\infty} \frac{\sqrt[n]{M(1)M(2)\cdots M(n)}}{M(n)} = e^{-1}.$$

Solution by Randy Schwartz, Schoolcraft College (retired), Ann Arbor, Michigan.

From the definition of the function M, we have $[M(n)+1]! > \alpha^n$ for $\alpha > 1$, so $\lim_{n\to\infty} M(n) = \infty$, and thus $\lim_{n\to\infty} \ln M(n) = \infty$. Also from the definition, we have

$$[M(n)]! \leq \alpha^n \Rightarrow \ln([M(n)]!) \leq n \ln \alpha \Rightarrow \frac{\ln([M(n)]!)}{n} \leq \ln \alpha,$$

and thus

$$\lim_{n\to\infty} \frac{\ln([M(n)]!)}{n} \leq \ln \alpha. \tag{1}$$

Applying Stirling's approximation to (1) leads to

$$\lim_{n\to\infty} \frac{\left(M(n)+\frac{1}{2}\right)\ln M(n) - M(n) + \frac{1}{2}\ln 2\pi}{n} \leq \ln \alpha$$

$$\lim_{n\to\infty} \left[\frac{M(n)}{n}(\ln M(n) - 1) + \frac{\ln M(n)}{2n} + \frac{\ln 2\pi}{2n}\right] \leq \ln \alpha$$

$$\lim_{n\to\infty} \left[\frac{M(n)}{n}(\ln M(n) - 1) + \frac{\ln M(n)}{2n}\right] \leq \ln \alpha$$

The last term inside the brackets is nonnegative and, from the foregoing, the factor $\ln M(n) - 1$ increases without bound; thus, $\frac{M(n)}{n}$ must vanish, since otherwise the above limit could not be a finite number such as $\ln \alpha$. Thus, we have established

$$\in_{n\to\infty} \frac{M(n)}{n} = 0.$$

We can deduce more the definition of the function M:

$$[M(n)+1]!\alpha^n$$

$$[M(n)+1]M(n)! > \alpha^n$$

$$[M(n)]! > \frac{\alpha^n}{M(n)+1}$$

$$\ln([M(n)!) > n\ln\alpha - \ln[M(n)+1]$$

$$\frac{\ln([M(n)!])}{n} > \ln\alpha - \frac{\ln[M(n)+1]}{n}$$

$$\lim_{n\to\infty} \frac{\ln([M(n)]!)}{n} \geq \ln \alpha,$$

and combining this with (1) yields

$$\lim_{n\to\infty} \frac{\ln)[M(n)!])}{n} = \ln \alpha$$

and then

$$\lim_{n\to\infty} \frac{\ln)[M(n)!])}{n} = 1. \tag{2}$$

Using Stirling again, we have

$$\lim_{n\to\infty} \frac{\ln([M(n)!)}{M(n)\ln M(n)} = \lim_{n\to\infty} \frac{\left(M(n)+\frac{1}{2}\right)\ln M(n) - M(n) + \frac{1}{2}\ln 2\pi}{M(n)\ln M(n)}$$

$$= \lim_{n\to\infty} \left[\frac{M(n)+\frac{1}{2}}{M(n)} - \frac{1}{\ln M(n)} + \frac{\ln 2\pi}{2M(n)\ln M(n)} \right]$$

$$= 1 - 0 + 0 = 1.$$

and combining this with (2) yields

$$\lim_{n\to\infty} \frac{M(n)\ln M(n)}{n\ln\alpha} = 1. \tag{3}$$

We can now calculate the requested value, L. We have

$$L = \lim_{n\to\infty} \frac{\sqrt[n]{\prod_{h=1}^{n} M(h)}}{M(n)} = \lim_{n\to\infty} \sqrt[n]{\prod_{h=1}^{n} \frac{M(h)}{M(n)}},$$

and then

$$\ln L = \lim_{n\to\infty} \sum_{h=1}^{n} \frac{1}{n} \ln\left[\frac{M(h)}{M(n)}\right].$$

There are many repeated terms in the above summation. The interval between $(j-1)!$ and $j!$, involving as it does a multiplication by j, encloses approximately $\log_\alpha j$ powers of α, each one of them associated with the same value of the function M. In other words, the number of integer solutions of $M(n) = j$ is asymptotically $\log_\alpha j = \frac{\ln j}{\ln\alpha}$. Using that as a weighting factor to gather the repeated terms, we can rewrite the above summation as

$$\ln L = \lim_{n\to\infty} \sum_{j=1}^{M(n)} \frac{1}{n} \cdot \frac{\ln j}{\ln\alpha} \ln\left[\frac{j}{M(n)}\right]$$

$$= \lim_{n\to\infty} \sum_{j=1}^{M(n)} \frac{(\ln j)^2 - \ln j \cdot \ln M(n)}{n\ln\alpha}$$

$$= \lim_{n\to\infty} \sum_{j=1}^{M(n)} \frac{(\ln j)^2 - \ln j \cdot \ln M(n)}{M(n)\ln M(n)}, \text{ using (3),}$$

and thus

$$\ln L = \lim_{n\to\infty} \left[\frac{1}{M(n)\ln M(n)} \sum_{j=1}^{M(n)} (\ln j)^2 - \frac{1}{M(n)} \sum_{j=1}^{M(n)} \ln j \right]. \tag{4}$$

Using inscribed and circumscribed rectangles, we have

$$\int_1^k \ln x\,dx < \sum_{j=1}^{k} \ln j < \int_2^{k+1} \ln x\,dx$$

$$\sum_{j=1}^{k} \ln j \approx \int_1^k \ln x \, dx = k \ln k - k + 1$$

$$\lim_{n\to\infty} \frac{1}{k} \sum_{j=1}^{k} \ln j = \ln k - 1,$$

and similarly

$$\sum_{j=1}^{k} (\ln j)^2 \approx \int_1^k (\ln x)^2 \, dx = k(\ln k)^2 - 2k \ln k + 2k - 2$$

$$\lim_{n\to\infty} \frac{1}{k \ln k} \sum_{j=1}^{k} (\ln j)^2 = \ln k - 2.$$

Applying these to (4) yields

$$\ln L = \lim_{n\to\infty} [(\ln M(n) - 2) - (\ln M(n) - 1)] = -1,$$

and thus

$$L = e^{-1}.$$

Also solved by Dmitry Fleischman, Santa Monica, CA; Lixing Han, U. of Michigan-Flint and Xinjia Tang, Chang Zhou U.; Albert Stadler, Herrliberg, Switzerland; and the proposer.

A recursively defined sequence of trigonometric functions

1188. *Proposed by Ángel Plaza, Universidad de Las Palmas de Gran Canaria, Las Palmas de Gran Canaria, Spain.*

Let $\{f_n(x)\}_{n\geq 1}$ be the sequence of functions recursively defined by $f_n(x) = \int_0^{f_{n-1}(x)} \sin t dt$, with initial condition $f_1(x) = \int_0^x \sin t dt$. For each $n \in \mathbb{N}$, find the value of p_n such that $L_n = \lim\limits_{x\to 0} \frac{f_n(x)}{x^{p_n}} \in \mathbb{R}\backslash\{0\}$ and the corresponding value L_n. Prove also that $\log_2(L_n^{-1}) = 3\log_2(L_{n-1}^{-1}) - 2\log_2(L_{n-2}^{-1})$ for $n \geq 3$.

Solution by Michael Vowe, Therwil, Switzerland.

We have

$$f_1(x) = \int_0^x \sin t \, dt = 1 - \cos x = \frac{x^2}{2!} + O\left(x^4\right)$$

and hence $p_1 = 2$, $L_1 = \frac{1}{2}$. Further

$$\begin{aligned} f_2(x) &= 1 - \cos(1 - \cos x) \\ &= \left(\frac{1-\cos x}{2!}\right)^2 - \left(\frac{1-\cos x}{4!}\right)^4 + \cdots = \frac{x^4}{2!4} + O\left(x^6\right), \end{aligned}$$

which means that $p_2 = 4$, $L_2 = \frac{1}{8}$.

Since

$$f_n(x) = 1 - \cos\left(f_{n-1}(x)\right),\ p_1 = 2,\ L_1 = \frac{1}{2},$$

we obtain

$$p_n = 2p_{n-1} = 2 \cdot 2p_{n-2} = \cdots = 2^{n-1}p_1 = 2^n$$

and

$$\begin{aligned} L_n &= \frac{1}{2!}\left(L_{n-1}\right)^2 = \frac{1}{2!} \cdot \frac{1}{(2!)^2}\left(L_{n-1}\right)^4 = \cdots = \frac{1}{2!^{1+2+4+\cdots+2^{n-2}}}\left(L_1\right)^{2^{n-1}} \\ &= \frac{1}{2^{2^{n-1}-1}} \cdot \frac{1}{2^{2^{n-1}}} = \frac{1}{2^{2^n-1}}. \end{aligned}$$

Now

$$\begin{aligned} 3\log_2\left(L_{n-1}^{-1}\right) - 2\log_2\left(L_{n-2}^{-1}\right) &= 3\left(2^{n-1}-1\right) - 2\left(2^{n-2}-1\right) \\ &= 2 \cdot 2^{n-1} - 1 = 2^n - 1 = \log_2 2^{2^n-1} = \log_2\left(L_n^{-1}\right). \end{aligned}$$

Also solved by Michel Bataille, Rouen, France; Brian Bradie, Christopher Newport U.; Paul Budney, Sunderland, MA; Hongwei Chen, Christopher Newport U.; Christopher Newport U. Problem Solving Seminar; Gerald Edgar, Denver, CO; Lixing Han, U. of Michigan-Flint; Justin Haverlick, State U. of New York at Buffalo; Eugene Herman, Grinnell C.; Christopher Jackson, Coleman, Florida; Elias Lampakis, Kiparissia, Greece; Albert Natian, Los Angeles Valley C.; Mark Sand, C. of Saint Mary; Randy Schwartz, Schoolcraft C. (retired); Albert Stadler, Herrliberg, Switzerland; Seán Stewart, Bomaderry, NSW, Australia; and the proposer. One incomplete solution and one incorrect solution were received.

A sum of harmonic sums

1189. *Proposed by Seán Stewart, Bomaderry, NSW, Australia.*

Evaluate the following sum:

$$\sum_{n=1}^{\infty} \frac{H_{n+1} + H_n - 1}{(n+1)(n+2)},$$

where $H_n = \sum_{k=1}^{n} \frac{1}{k}$ denotes the nth harmonic number.

Solution by Robert Agnew, Palm Coast, Florida.

The sum

$$S = \sum_{n=1}^{\infty} \frac{H_{n+1} + H_n - 1}{(n+1)(n+2)}$$

can be written as

$$S = \sum_{n=1}^{\infty} \frac{1}{(n+1)(n+2)}\left(-1 + \frac{1}{n+1} + 2 \cdot \sum_{k=1}^{n} \frac{1}{k}\right)$$

$$= -\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+2)} + \sum_{n=1}^{\infty} \frac{1}{(n+1)^2(n+2)} + 2 \cdot \sum_{n=1}^{\infty} \frac{1}{(n+1)(n+2)} \sum_{k=1}^{n} \frac{1}{k}.$$

Evaluating each of these sums in turn gives

$$\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+2)} = \sum_{n=1}^{\infty} \left(\frac{1}{n+1} - \frac{1}{n+2} \right) = \frac{1}{2};$$

$$\begin{aligned} \sum_{n=1}^{\infty} \frac{1}{(n+1)^2(n+2)} &= \sum_{n=1}^{\infty} \left(-\frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{(n+1)^2} \right) \\ &= -\sum_{n=1}^{\infty} \left(\frac{1}{n+1} - \frac{1}{n+2} \right) + \sum_{n=1}^{\infty} \frac{1}{(n+1)^2} \\ &= -\frac{1}{2} + \left(\frac{\pi^2}{6} - 1 \right) \\ &= -\frac{3}{2} + \frac{\pi^2}{6}; \end{aligned}$$

and

$$\begin{aligned} \sum_{n=1}^{\infty} \frac{1}{(n+1)(n+2)} \sum_{k=1}^{n} \frac{1}{k} &= \sum_{k=1}^{\infty} \frac{1}{k} \sum_{n=k}^{\infty} \frac{1}{(n+1)(n+2)} \\ &= \sum_{k=1}^{\infty} \frac{1}{k} \sum_{n=k}^{\infty} \left(\frac{1}{n+1} - \frac{1}{n+2} \right) \quad = \sum_{k=1}^{\infty} \frac{1}{k(k+1)} \\ &= 1. \end{aligned}$$

Hence

$$S = \frac{\pi^2}{6}.$$

Also solved by Arkady Alt, San Jose, CA; Farrukh Rakhimjanovich Ataev, Westminster International U., Tashkent, Uzbekistan; Michel Bataille, Rouen, France; Necdet Batir, Nevşehir Haci Bektaş Veli U.; Khristo Boyadzhiev, Ohio Northern U.; Paul Bracken, U. of Texas, Edinburg; Brian Bradie, Christopher Newport U.; Hongwei Chen, Christopher Newport U.; Geon Choi, Yonsei U.; Nandan Sai Dasireddy, Hyderabad, India; Bruce Davis, St. Louis Comm. C. at Florissant Valley; Giuseppe Fera, Vicenza, Italy; Subhankar Gayen, West Bengal, India; Michael Goldenberg, Baltimore Polytechnic Inst. and Mark Kaplan, U. of Maryland Global Campus; GWStat Problem Solving Group; Lixing Han, U. of Michigan - Flint and Xinjia Tang, Chang Zhou U.; Eugene Herman, Grinnell C.; Walther Janous, Innsbruck, Austria; Kee-Wai Lau, Hong Kong, China; Seungheon Lee, Yonsei U.; Graham Lord, Princeton, NJ; Missouri State U. Problem Solving Group; Shing Hin Jimmy Pa; Ángel Plaza and Francisco Perdomo, Universidad de Las Palmas de Gran Canaria, Las Palmas, Spain; Rob Pratt, Apex, NC; Arnold Saunders, Arlington, VA; Volkhard Schindler, Berlin, Germany; Randy Schwartz, Schoolcraft C. (retired); Allen Schwenk, Western Michigan U. Albert Stadler, Herrliberg, Switzerland; Marián Ŝtofka, Slovak U. of Technology; Enrique Treviño, Lake Forest C.; Michael Vowe, Therwil, Switzerland; and the proposer.

A second-order differential equation

1190. *Proposed by George Stoica, Saint John, New Brunswick, Canada.*

Find all twice differentiable functions $y = y(x)$ such that $(y + x)y'' = y'(y' + 1)$.

Solution by Eugene Herman, Grinnell College, Grinnell, Iowa.

Substituting $z(x) = y(x) + x$ into the differential equation yields $zz'' = (z' - 1)z'$. This has solutions $z = k$ and $z = x + k$. Other than these, we have

$$\frac{d}{dx}\left(\frac{z' - 1}{z}\right) = \frac{zz'' - (z' - 1)z'}{z^2} = 0$$

and so $z' - 1 = cz$, where $c \neq 0$. Separating variables yields $z = \frac{ke^{cx} - 1}{c}$. Therefore, the solutions for y are

$$k - x, \quad k, \quad \frac{ke^{cx} - 1}{c} - x \text{ (where } c \neq 0).$$

Editor's note: Solvers exercised various degrees of care in ensuring the existence of an interval on which one could safely avoid dividing by zero. In the interests of space, we have not incorporated that analysis here.

Also solved by Robert Agnew, Palm Coast, FL; Arkady Alt, San Jose, CA; Tomas Barajas, U. of Arkansas at Little Rock; Michel Bataille, Rouen, France; Paul Bracken, U. of Texas, Edinburg; Brian Bradie, Christopher Newport U.; Hongwei Chen, Christopher Newport U.; Richard Daquila, Muskingham U.; Bruce Davis, St. Louis Comm. C. at Florissant Valley; Michael Goldenberg, Baltimore Polytechnic Inst. and Mark Kaplan, U. of Maryland Global Campus; Anna DePoyster, Missie Bogard, Rylee Buck, and Chanty Gray, (students) U. of Arkansas, Little Rock; Raymond Greenwell, Hofstra U.; Lixing Han, U. of Michigan-Flint and Xinjia Tang, Chang Zhou U.; Justin Haverlick, State U. of New York at Buffalo; Logan Hodgson; Walther Janous, Innsbruck, Austria; Harris Kwong, SUNY Fredonia; Seungheon Lee, Yonsei U.; William Littlejohn, Jason Pearson, and Cole Stillman (students) U. of Arkansas, Little Rock; James Magliano, Union Country C. (emeritus); Albert Natian, Los Angeles C.; Randy Schwartz, Schoolcraft C. (retired); Albert Stadler, Herrliberg, Switzerland; Seán Stewart, Bomaderry, NSW, Australia; Nora Thornber, Raritan Valley Comm. C.; and the proposer.