Problem Solving Session (aka MFD prep)
CSU Fresno
March 14, 2015
Topics: The number π, Area and Volume

1. (MH 2014 11-12) Triangle ABC has $AC = 15$, $BC = 13$, and $AB = 4$. What is the length of the altitude from C to the extension of AB?
   (a) 9
   (b) 10
   (c) 11
   (d) 12
   (e) 13
2. (MH 2014 11-12) Points A, C, and D lie on a circle. Point B lies outside the circle such that B, D, and C are collinear with D between B and C, and BA is tangent to the circle. If $AB = 2$, $AC = 3$, and $BD = 1$, what is the area of triangle ABC?
   (a) 1
   (b) 2
   (c) $\frac{3}{4}\sqrt{15}$
   (d) $\frac{3}{4}\sqrt{11}$
   (e) $2\sqrt{11}$
3. (MH 2014 11-12) A right circular cone has height equal to radius. What is the ratio of its volume to that of a cube inscribed inside it, with the base of the cube lying on the base of the cone?
   (a) $\frac{\pi}{12}(10 + \sqrt{2})$
   (b) $\frac{\pi}{12}(10 + 3\sqrt{2})$
   (c) $\frac{\pi}{12}(10 + 5\sqrt{2})$
   (d) $\frac{\pi}{12}(10 + 7\sqrt{2})$

(e) $\frac{\pi}{12}(10 + 9\sqrt{2})$

4. (MH 2014 11-12) $ABCD$ is a rectangle in which the length AB minus the length AD equals 10. Inside $ABCD$ is a square $WXYZ$ with sides parallel to those of the rectangle, and W closest to A, and X closest to B. The total of the areas of the trapezoids $XBCY$ and $AWZD$ is 1000, while the total area of the trapezoids $ABXW$ and $ZYCD$ is 400. What is the area of the square $WXYZ$?

   (a) 400

   (b) 1600

   (c) 3600

   (d) 4900

   (e) 6400

5. (MH 2014 11-12) Let BE be a median of triangle ABC, and let D be a point on AB such that $BD/DA = 3/7$. What is the ratio of the area of triangle BED to that of triangle ABC?

   (a) 3/20

   (b) 7/20

   (c) 1/5

   (d) 1/4

   (e) the answer cannot be determined

6. (MH 2014 9-10) Square $ABCD$ has side length 10. If point E is on $\overline{BC}$, and the area of $\triangle ABE$ is 40, what is BE?

   (a) 4

   (b) 5

   (c) 6

   (d) 8

7. (MH 2014 9-10) If a and b are the length of the legs of a right triangle whose hypotenuse is 10 and whose area is 20, find $(a + b)^2$.

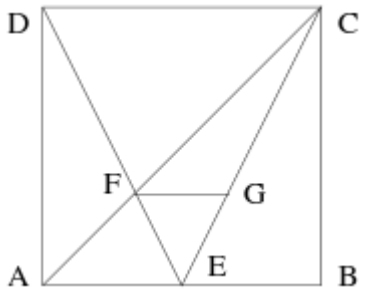
(a) 180

(b) 140

(c) 120

(d) 100

8. (LF 2014 11-12) The square $ABCD$ has sides of length 2. Point E is the midpoint of edge AB. Point F is the intersection of lines AC and DE. Line FG is parallel to line AB. The area of $\triangle EFG$ is:



(a) $\frac{2}{3}$

(b) $\frac{1}{3}$

(c) $\frac{2}{9}$

(d) $\frac{4}{9}$

(e) None of the above

9. (LF 2014 9-10) A cube of ice has melted so that its surface area has decreased by 19%. Assuming that at all times, the cube maintains length = width = height, by what percentage has the volume decreased?

(a) 26.7%

(b) 26.9%

(c) 27.1%

(d) 27.3%

(e) None of these

10. (LF 2014 9-10) In $\triangle ABC$, point D lies on the side AB. The length of AB is 10 feet and the length of AD is x feet. What is the value of x such that the area enclosed by ADC

is twice the area enclosed by $\triangle BDC$?

(a) $x = 8$

(b) $x = \frac{9}{2}$

(c) $x = 5$

(d) $x = \frac{20}{3}$

(e) None of these

11. (LF 2014 9-10) The sum of the first 2014 positive odd integers is subtracted from the sum of the first 2014 positive even integers. What is the result?

(a) 1

(b) 0

(c) 4028

(d) 2013

(e) None of these

12. (LF 2014 9-10) Two cubes (length = width = height) have respective volumes V_1 and V_2 that satisfy $V_1/V_2 = 10$. Let S_1 and S_2 be the respective surface areas of the cubes, i.e. S_1 corresponds to V_1 and S_2 corresponds to V_2. Determine the ratio of surface areas S_1/S_2.

(a) $S_1/S_2 = \sqrt[3]{150}$

(b) $S_1/S_2 = \sqrt[3]{10}$

(c) $S_1/S_2 = \sqrt[3]{200}$

(d) $S_1/S_2 = \sqrt[3]{100}$

(e) None of these

13. (MH 2014 9-10) The largest area of a triangle that can be inscribed in a semicircle of radius r is

(a) $2r^2$

(b) r^2

(c) $\frac{1}{2}r^2$

(d) $\frac{1}{4}r^2$

14. (MH 2014 9-10) A gold bar is a rectangular solid measuring $2 \times 3 \times 4$. It is melted down, and three cubes of equal size are constructed from this mold. What is the length of a side of each cube?

    (a) 8

    (b) 6

    (c) 4

    (d) 2

15. (MH 2014 9-10) A right circular cylinder has a radius of 8 and height of π^2. If a cube has the same volume as the cylinder, what is the length of an edge of the cube?

    (a) $4\sqrt{\pi}$

    (b) $8\sqrt{\pi}$

    (c) $4\pi\sqrt{\pi}$

    (d) 4π

16. (LF 2013 11-12) One sphere is inscribed in a cube, while the cube is also inscribed in another sphere. Find the ratio of the volumes of the larger sphere to the smaller sphere.

    (a) $\sqrt{3}$

    (b) $2\sqrt{3}$

    (c) $3\sqrt{3}$

    (d) $3\sqrt{2}$

    (e) None of the above

17. (LF 2013 9-10) A circle is inscribed in the isosceles triangle with side lengths 6, 6 and 4. Determine the area of the inscribed circle.

    (a) $\frac{\pi}{2}$

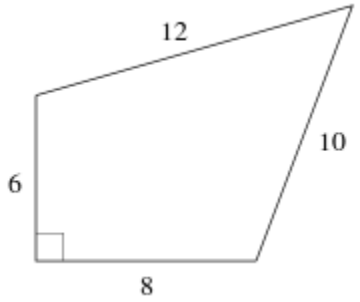
    (b) $\frac{3\pi}{2}$

    (c) $\frac{5\pi}{2}$

    (d) $\frac{7\pi}{2}$

(e) None of these

18. (MH 2012 11-12) Find the area of the quadrilateral shown below.



(a) 48

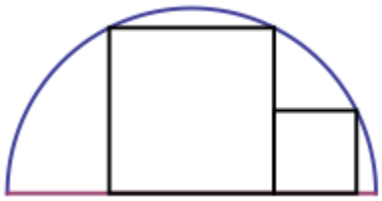
(b) 64

(c) 72

(d) 80

(e) None of the above

19. (LF 2012 9-12) In the figure below, the semicircle has radius equal to 1 inch, and the two adjacent squares are inscribed as pictured. What is the area of the smaller square?



(a) Area $= \frac{1}{4}$ in^2

(b) Area $= \frac{1}{\sqrt{5}}$ in^2

(c) Area $= \frac{2}{\sqrt{7}}$ in^2

(d) Area $= \frac{1}{1+\sqrt{5}}$ in^2

(e) None of these

20. (LF 2011 9-12) Lenny melts 2011 $1''$ by $1''$ by $1''$ ice cubes and refrezes the water to form one large ice cube (all side lengths equal). The side length of the large cube is

(a) between 10 and 11 inches.

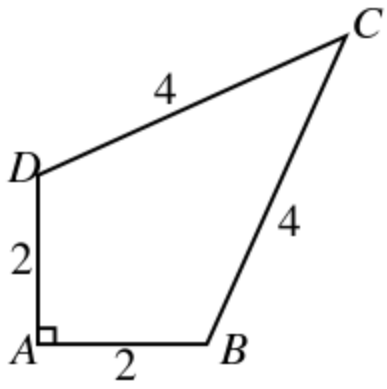
(b) between 11 and 11 inches.

(c) between 12 and 13 inches.

(d) between 13 and 14 inches.

(e) None of these

21. (LF 2011 9-12) In the figure below, the lengths are as labeled and the angle at A is a right angle. The area enclosed by $ABCD$ is ...



(a) $4 + 2\sqrt{7}$

(b) $2 + 2\sqrt{6}$

(c) $2 + 2\sqrt{7}$

(d) $4 + \sqrt{6}$

(e) None of these