





**Solution.** (d) Thirty percent off of 78 is  $0.7 \times 78 = 54.60$ . Add in the 10% sales tax,  $54.60 + 5.46 = 60.06$ . The change you get back is  $\$100 - \$60.06 = \$39.94$ .

4. A cube of ice has melting so that its surface area has decreased by 19%. Assuming at all times, the cube maintains length = width = height, by what percentage has the volume decreased?
- (a) 26.7% (b) 26.9%
- (c) 27.1% (d) 27.3%
- (e) None of these

**Solution.** (c) Denote the side length of the cube by  $x$  before melting and by  $y$  after melting. Then the two surfaces areas are  $6x^2$  and  $6y^2$ , respectively. We are assuming  $6y^2 = (1 - .19)6x^2 = .81 \times 6x^2$ . This implies  $y = \sqrt{.81}x = .9x$ . Cubing each side gives us the volume,  $y^3 = .729x^3$ . So the volume has decreased by a factor of  $1 - .729 = .271$ . This means, the volume has decreased by 27.1%.

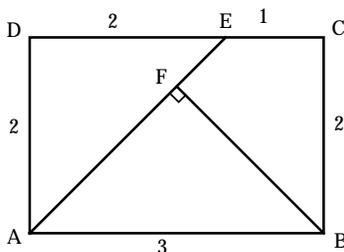
5. The 9-digit number  $N = 1234d4321$  is divisible by 9. What is the value of the digit  $d$ ?
- (a) 7 (b) 6
- (c) 5 (d) 4
- (e) None of these

**Solution.** (a) A number is divisible by 9 precisely when its digits add up to a multiple of 9. The sum of the digits of  $N$  is

$$1 + 2 + 3 + 4 + d + 4 + 3 + 2 + 1 = 20 + d.$$

In order for  $20 + d$  to be divisible by 9, we must have  $d = 7$ . And indeed,  $123474321 = 9 \times 13719369$

6. In the figure below  $ABCD$  is a rectangle with indicated side lengths. Also,  $\overline{BF} \perp \overline{AE}$ . Find the length of  $BF$ .



- (a)  $BF = 2\sqrt{3}$                       (b)  $BF = \sqrt{5}$   
(c)  $BF = \frac{5}{2}$                       (d)  $BF = \frac{3\sqrt{2}}{2}$   
(e) None of these

**Solution.** (d) Since  $m\angle ABF = 90 - m\angle BAF = m\angle DAE$ , we have similar triangles  $\triangle BAF \sim \triangle AED$ . Hence,

$$\frac{BF}{AB} = \frac{AD}{EA}.$$

Now,  $AB = 3$ ,  $AD = 2$  and by the Pythagorean Theorem applied to  $\triangle ADE$  we have  $EA = 2\sqrt{2}$ . This gives us,

$$\frac{BF}{AB} = \frac{AD}{EA} \implies \frac{BF}{3} = \frac{2}{2\sqrt{2}} \implies BF = \frac{3\sqrt{2}}{2}.$$

7. Find the value of  $a$  so that the two lines  $2013x + 2012y = 1$  and  $2014x + ay = 1$  are mutually perpendicular.

- (a)  $-\frac{2027091}{1009}$                       (b)  $-\frac{2027091}{1008}$   
(c)  $-\frac{2027091}{1007}$                       (d)  $-\frac{2027091}{1006}$   
(e) None of these

**Solution.** (d) The slopes of mutually perpendicular lines are negative reciprocals of one another. We find the slopes by solving for  $y$  in each





**Solution. (d)** Let  $n$ ,  $d$  and  $q$  be the original number of nickels, dimes and quarters in Lenny's purse. Since there are 45 coins, we have our first equation:

$$\text{Eq}_1 : n + d + q = 45.$$

The dollar amount is \$4, so this gives us another equation  $5n + 10d + 25q = 400$ . Divide by 5 to get equation number 2,

$$\text{Eq}_2 : n + 2d + 5q = 80.$$

After, the switch, we have  $25n + 10d + 5q = 800$  and upon dividing by 5, we have

$$\text{Eq}_3 : 5n + 2d + q = 160.$$

Solve the above three equations in 3 unknowns, to get  $n = 25$ ,  $d = 15$  and  $q = 5$ . So, Lenny originally had 5 quarters.