Problems on Vieta's formulas

1. For how many real numbers a does the quadratic equation $x^2 + ax + 123 = 0$ have two integer roots?

2. Let a, b, c, and d be four distinct one-digit numbers. What is the maximum possible value of the sum of the roots of the equation $(x - a)(x - b) + (x - c)(x - d) = 0$?

3. The sum of the zeros, the product of the zeros, and the sum of the coefficients of the function $f(x) = ax^2 + bx + c$ are equal. Their common value must also be which of the following?

   (a) the coefficient of x^2
   (b) the coefficient of x
   (c) the y-intercept of the graph $y = f(x)$
   (d) one of the x-intercepts of the graph of $y = f(x)$
   (e) the mean of the x-intercepts of the graph of $y = f(x)$

4. The quadratic equation $x^2 + mx + n = 0$ has roots twice those of $x^2 + px + m = 0$, and none of m, n, and p is zero. What is the value of n/p?

5. Let a and b be the roots of the equation $x^2 - mx + 2 = 0$. Suppose that $a + \frac{1}{b}$ and $b + \frac{1}{a}$ are the roots of the equation $x^2 - px + q = 0$. What is q?

6. What is the sum of the reciprocals of the roots of the equation $\frac{2003}{2004}x + 1 + \frac{1}{x} = 0$?

7. If α and β are solutions of $x^2 + px + q = 0$, find $\alpha^2 + \beta^2$ in terms of p and q.

8. Let a and b be the roots of $x^2 - 3x - 1 = 0$. Find a quadratic equation whose roots are a^2 and b^2.

9. Suppose the roots of $x^3 + 3x^2 + 4x - 11 = 0$ are a, b, and c, and the roots of $x^3 + rx^2 + sx + t = 0$ are $a + b$, $b + c$, and $c + a$. Find r. (A harder problem: find t.)

10. Three of the roots of $x^4 + ax^2 + bx + c = 0$ are 2, -3, and 5. Find the values of a, b, and c.