

Section 4.1, Problem 22:

Is it possible to define a quadratic polynomial whose graph contains the four points $(-1, -2)$, $(0, -2)$, $(1, 0)$, and $(2, 2)$?

Solution:

Suppose such a quadratic polynomial $ax^2 + bx + c$ exists. Then, plugging in the above x -values and setting the value of the polynomial to the corresponding y -values, we have:

$$a - b + c = -2,$$

$$c = -2,$$

$$a + b + c = 0,$$

$$4a + 2b + c = 2.$$

Since $c = -2$, the other three equations become:

$$a - b = 0,$$

$$a + b = 2,$$

$$4a + 2b = 4.$$

Now adding the first two of these gives $2a = 2$, so $a = 1$, then $b = 1$, however, the third equation gives $4 + 2 = 4$, a contradiction.

Therefore there is no such quadratic polynomial.