Math 151

Fall 2008

Test 2

Name: ______________________________

This test is due Wed, 11/05/08, at the beginning of the class period (9:15 am).

You may use your notes, the textbook for this class, and any materials posted on the course web page (such as homework solutions, practice test solutions, etc.). In fact, you are encouraged to do so whenever you are not sure about a notation or feel that you have to review a definition or examples, etc. The goal is to learn the material well. You may use a calculator, but it probably won't be too helpful.

You may not use any other books, any materials other than the ones posted on the course web page, someone else's notes, etc. Please no joint work. Moreover, please do not even discuss the problems with anyone. (Limit all conversations to something of the sort "Have you finished the test yet?" – "Yes, I finished it last night".)

1. (12 points total)

   (a) (10 points) Find the order of each group, whether it is abelian, and whether it is cyclic. Provide brief explanations (you may refer to a theorem or an example in the book).

group	order	abelian?	cyclic?
$\mathbb{Z}_{12}$			
$\mathbb{Z}_3 \times \mathbb{Z}_4$			
$\mathbb{Z}_2 \times \mathbb{Z}_6$			
$\mathbb{Z}_{12}^{\times}$			
S_{12}			
A_4			
$GL_{12}(\mathbb{R})$			

   (b) (2 points) Are any of the above groups isomorphic? (Explain.)

2. (5 points total) Let $\phi(\mathbb{Z}_2 \times \mathbb{Z}_4) \to \mathbb{Z}_2$ be defined by $\phi([a]_2, [b]_4) = [a + b]_2$.

(a) (3 points) Show that ϕ is a homomorphism.

(b) (2 points) Find the kernel of ϕ.

3. (3 points) Prove that $\mathbb{Z}$ is a normal subgroup of $\mathbb{R}$.

4. (4 points total) In D_6, let a and b denote the counterclockwise rotation through an angle of 60 degrees and the flip about the vertical line, respectively.

   (a) (2 points) Give a geometric description of the rigid motion ab. (Is it a rotation? If so, through what angle? Or is it a flip? If so, about what line? Or is it some other rigid motion?)

   (b) (2 points) What is the order of ab?

5. (6 points total) Let G be any group. Define a function $\phi : G \to G$ by $\phi(x) = x^{-1}$ for all $x \in G$.

(a) (3 points) Prove that ϕ is one-to-one and onto.

(b) (3 points) Give an example of a group G for which the function ϕ defined above is an isomorphism (and prove that it is).

Optional (for extra credit, 3 points): Prove that any group of order 24 contains at least one element of order 2.