

Practice problems for Test 2

Answers

1. (Note: feel free to show me your examples to make sure they are correct.)

| group | order | abelian? | cyclic? |
|------------------------------------|----------|----------|---------|
| \mathbb{Z}_5^* | 4 | yes | yes |
| \mathbb{Z}_6 | 6 | yes | yes |
| S_3 | 6 | no | no |
| $\mathbb{Z}_4 \times \mathbb{Z}_2$ | 8 | yes | no |
| \mathbb{Z} | ∞ | yes | yes |
| $GL_2(\mathbb{R})$ | ∞ | no | no |
| $\{e\}$ =trivial | 1 | yes | yes |
| D_5 | 10 | no | no |
| $Mat_{2 \times 3}(\mathbb{Z}_2)$ | 64 | yes | no |
| \mathbb{R} | ∞ | yes | no |

- 3.
- $\mathbb{R} \cong \mathbb{R}^+$
- ;
- $\mathbb{Z}_2 \times \mathbb{Z}_8 \cong \mathbb{Z}_8 \times \mathbb{Z}_2$
- .

5. 4. $\left\{ \left[\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right], \left[\begin{array}{cc} 2 & 0 \\ 0 & 3 \end{array} \right], \left[\begin{array}{cc} 4 & 0 \\ 0 & 4 \end{array} \right], \left[\begin{array}{cc} 3 & 0 \\ 0 & 2 \end{array} \right] \right\}$.

6. (a) 8

(b) $H = \{0, 6, 12, 18\}$ is a cyclic subgroup. Generators: 6 and 18. $K = \{0, 4, 8, 12, 16, 20\}$ is a cyclic subgroup. Generators: 4 and 20. $H \cap K = \{0, 12\}$ is a cyclic subgroup. Generator: 12. $H \cup K = \{0, 4, 6, 8, 12, 16, 18, 20\}$ is not a subgroup. $H + K = \{0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22\}$ is a cyclic subgroup. Generators: 2, 10, 14, 22.

7. (a) Yes.
- $\text{Ker}(f) = \{0\}$
- . Image =
- $3\mathbb{Z}$
- . One-to-one. Not onto. Not isomorphism.
-
- (b) Yes.
- $\text{Ker}(f) = 4\mathbb{Z}$
- . Image =
- \mathbb{Z}_4
- . Not one-to-one. Onto. Not isomorphism.
-
- (c) Yes.
- $\text{Ker}(f) = 3\mathbb{Z}$
- . Image =
- $2\mathbb{Z}_6$
- . Not one-to-one. Not onto. Not isomorphism.
-
- (d) No.
-
- (e) Yes.
- $\text{Ker}(f) = \{[0]\}$
- . Image =
- \mathbb{Z}_{10}
- . One-to-one. Onto. Isomorphism.
-
- (f) Yes.
- $\text{Ker}(f) = \{(x, -x)\}$
- . Image =
- \mathbb{R}
- . Not one-to-one. Onto. Not isomorphism.
-
- (g) Yes.
- $\text{Ker}(f) = \{(1, 1)\}$
- . Image =
- $\left\{ \left[\begin{array}{cc} a & b \\ -2b & a + 3b \end{array} \right] \right\}$
- . One-to-one. Not onto. Not isomorphism.