Math 151

Practice problems for Test 3 - Answers

1. $q(x) = x^3 - 2x$, $r(x) = 7x + 1$
2. (a) $d(x) = x^3 + 4x^2 + 5x + 2$
   (b) $a(x) = 5$, $b(x) = 2x + 1$
3. $[x + 4]^{-1} = 3x^2 + 3x + 1$
4. -2 and -4
5. over $\mathbb{Z}$: $x^3 - 2$ is irreducible
   over $\mathbb{Q}$: still irreducible
   over $\mathbb{R}$: $\left(x - \sqrt[3]{2}\right)\left(x^2 + \sqrt[3]{2}x + \sqrt[3]{4}\right)$
   over $\mathbb{C}$: $\left(x - \sqrt[3]{2}\right)\left(x + \dfrac{\sqrt[3]{2} + \sqrt[3]{2}\sqrt{3}i}{2}\right)\left(x + \dfrac{\sqrt[3]{2} - \sqrt[3]{2}\sqrt{3}i}{2}\right)$
   over $\mathbb{Z}_3$: factors as $(x+1)^3$, so $x+1$ is the only irreducible factor (up to constant)
6. $x^3 + x + 1$ and $x^3 + x^2 + 1$
7. Use $p = 5$.
8. Yes. No. No.
9. (a) $(1,1)$, $(1,3)$, $(1,5)$, $(1,7)$, $(5,1)$, $(5,3)$, $(5,5)$, $(5,7)$
   (b) $(\pm 1, \pm 1, \pm 1)$ (8 elements total)
   (c) all elements of the form (a, b) where $a \neq 0$, $b \neq 0$
10. (a) $\phi(x) = x$.
    (b) None.
11. (a) Yes.
    (b) Yes.
    (c) No.
    (d) No.