Math 250 **Maria Nogin**

Practice test 2 - Answers

1. The limit along the x-axis is $\lim_{(x,0)\to(0,0)} \frac{x(x-0)}{x^2+0^2} = 1$, but the limit along the y-axis is $\lim_{(0,y)\to(0,0)} \frac{0(0-y)}{0^2+y^2} = 0$. Since these two limits are not equal, the limit $\lim_{(x,y)\to(0,0)} \frac{x(x-y)}{x^2+y^2}$ does not exist.

2. $f_x(x,y) = \cos x + ye^{xy}, \ \ f_y(x,y) = -\sin y + xe^{xy};$
$f_{xx}(x,y) = -\sin x + y^2e^{xy}, \ \ f_{yy}(x,y) = -\cos y + x^2e^{xy}, \ \ f_{xy} = f_{yx} = e^{xy} + xye^{xy}$

3. (a) $z - 3 = \frac{1}{6}(x-8) + \frac{1}{3}(y-1)$, or $z = \frac{x}{6} + \frac{1}{3}y + \frac{4}{3}$
   (b) $L(x,y) = \frac{x}{6} + \frac{1}{3}y + \frac{4}{3}$

4. (a) $2\left(\ln(x+y) + \frac{x}{x+y}\right)t + \frac{3xt^2}{x+y}$
   (b) $\frac{\partial z}{\partial t} = \frac{4t^3}{1+x^2} - 3y^2s, \ \ \frac{\partial z}{\partial s} = \frac{2}{1+x^2} - 3y^2t$
   (c) 12

5. (a) $< \sqrt{y}, \frac{x}{2\sqrt{y}} >$
   (b) $< 2, \frac{3}{4} >$
   (c) $\frac{5}{4}$
   (d) the maximum rate of change of f is $\frac{\sqrt{73}}{4}$, occurs in the direction of the unit vector $< \frac{8}{\sqrt{73}}, \frac{3}{\sqrt{73}} >$

6. (a) $(-1,-1)$, $(-1,1)$, $(1,-1)$, $(1,1)$
   (b) a local maximum value is $f(-1,-1) = 4$, a local minimum value is $f(1,1) = -4$
   (c) none
   (d) the absolute maximum value is $f\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right) = \frac{5}{\sqrt{2}}$;
   the absolute minimum value is $f\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = -\frac{5}{\sqrt{2}}$
   (e) same as in (d)

7. (a) approximately 22.5 (Note: one reasonable approach is to use the Midpoint Rule with $m = 2$ or $m = 4$, and $n = 3$ or $n = 6$; larger values of m and n would be too much work, but would not give a much better estimate since we do not know precise values of f.)
   (b) approximately 1.9

8. (a) 6
   (b) 192
   (c) $\dfrac{7}{6}$
   (d) $\dfrac{(1 - \cos 9)\pi}{4}$

9. $m = \dfrac{\pi}{2}$; $(\overline{x}, \overline{y}) = \left(\dfrac{\pi^2 - 8}{2\pi - 4}, \dfrac{\pi + 2}{16} \right)$

10. 8π