Practice final

The Final Exam is on Monday, December 11, from 10:30 AM - 12:30 PM. The exam is comprehensive, so review everything we studied in this class. The actual test will consist of 8-10 problems. A good way to study for the final would be:

- Do the (optional) Review problem set (on WeBWorK).
- Review homework assignments (both on WeBWorK and from the book, including the recommended problems). Identify areas you were (or still are) having trouble with and review those.
- Review practice tests, tests, and quizzes. If you are not sure how to do some problems, review the corresponding material.
- Prepare your notes: one letter size sheet (both sides) of notes is allowed on the final.
- Do the problems given below to check your readiness. If needed, review again.
- Answers to these problems will be available. Check yours.
- Do more problems (e.g. from the book). In fact, do as many problem as you have time to. Always check your answers.
- Don't study the night before the exam: get some rest and sleep.
- Good luck!

"Train hard, fight easy"
Alexander V. Suvorov, Russian Field Marshal, 1729-1800

1. Describe in words the surface/region/solid in $\mathbb{R}^3$ represented by the equation of inequality.
   (a) $y = z$
   (b) $x^2 + y^2 = 4$
   (c) $\rho > 9$ (spherical coordinates)
   (d) $\theta = \pi$ (cylindrical or spherical coordinates)
   (e) $z = 8 - 4x^2 - 4y^2$
   (f) $r(u, v) = (1 + 2u)\mathbf{i}+(-u + 3v)\mathbf{j}+(2 + 4u + 5v)\mathbf{k}$
2. Find a vector that has the same direction as $< 1, 2, 3 >$ but has length 6.

3. Let $a =< 3, 4, 0 >$, $b =< 0, -1, 2 >$. Find the following:

   (a) $a \cdot b$,

   (b) $a \times b$,

   (c) the angle between a and b,

   (d) the area of the parallelogram determined by a and b.

4. Find both parametric and symmetric equations of the line through the point $(1, 0, -3)$ and parallel to the vector $< 2, -4, 5 >$.

5. Find an equation of the plane through the origin and the points $(2, -4, 6)$ and $(5, 1, 3)$.

6. Sketch the curve with the vector equation $r(t) =< 1 + t, 3t^2 - 2 >$. Find $r'(t)$. Sketch $r'(1)$.

7. Find the length of the curve given by $r(t) = \mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$, $0 \le t \le 1$.

8. Find the velocity, acceleration, and speed of a particle with the position function $r(t) =< \sin t, 2\cos t >$. Sketch the path of the particle and draw the velocity and acceleration vectors for $t = 0$.

9. Let $f(x, y) = \sqrt{x + 2y}$.

   (a) Find and sketch the domain of f.

   (b) Sketch a contour map of f.

   (c) Find f_x, f_y, f_{xx}, f_{yy}, and f_{xy}.

   (d) Find ∇f.

   (e) Find $D_u f(1, 4)$ in the direction of the vector $u =< -1, 1 >$.

   (f) Find the tangent plane approximation of f at $(1, 4)$.

   (g) Find the maximum and minimum values of f on the circle $(x - 1)^2 + (y - 1)^2 = 1$.

10. Describe the domain and the level surfaces of $f(x, y, z) = x^2 + y^2 - z$.

11. Show that the limit $\lim_{(x,y)\to(0,0)} \dfrac{6x^3y}{2x^4 + y^4}$ does not exist.

12. Calculate the following integrals:

    (a) $\displaystyle\int_0^5 \int_{-1}^3 2 \, dx \, dy$,

    (b) $\displaystyle\iint_D xy \, dA$, where D is the triangle with vertices $(1, 0)$, $(0, 1)$, and $(1, 1)$,

(c) $\displaystyle\iint_R (x+y)\, dA$, where R is the region that lies to the left of the y-axis between the circles $x^2+y^2=1$ and $x^2+y^2=4$,

(d) $\displaystyle\int_C ye^x\, ds$, where C is the line segment joining $(1,2)$ to $(4,7)$,

(e) $\displaystyle\iint_S y\, dS$, where S is the surface $z=\frac{2}{3}(x^{3/2}+y^{3/2})$, $0 \le x \le 1$, $0 \le y \le 1$.

13. Find the area of the part of the sphere $x^2+y^2+z^2=4$ that lies above the plane $z=1$.

14. Find the volume of the solid enclosed by the paraboloid $x=y^2+z^2$ and the plane $x=16$.

15. Find the Jacobian of the transformation given by $x=u^2-v^2$, $y=u^2+v^2$.

16. Sketch the vector field $\mathbf{F}(x,y)=(x+y)\mathbf{i}$.

17. Show that $\mathbf{F}$ is a conservative vertor field. Find a function f such that $\mathbf{F}=\nabla f$.

    (a) $\mathbf{F}=e^y\mathbf{i}+xe^y\mathbf{j}$

    (b) $\mathbf{F}=2xy\mathbf{i}+(x^2+2yz)\mathbf{j}+y^2\mathbf{z}$

18. Use Green's Theorem to evaluate the line integral along the given positively oriented curve: $\displaystyle\int_C x^2y^2dx+4xy^3dy$, where C is the triangle with vertices $(0,0)$, $(1,3)$, and $(0,3)$.

19. Find the curl and the divergence of the vector field $\mathbf{F}=x^2yz\mathbf{i}+xy^2z\mathbf{j}+xyz^2\mathbf{z}$.

20. Use Stokes' Theorem to evaluate $\displaystyle\int_C \mathbf{F}\cdot d\mathbf{r}$, where $\mathbf{F}(x,y,z)=e^{-x}\mathbf{i}+e^x\mathbf{j}+e^z\mathbf{z}$, and C is the boundary of the part of the plane $2x+y+2z=2$ in the first octant, oriented counterclockwise as viewed from above.