MATH 250
Test 1 Solutions

1. Let $v =< 9, 5, 1 >$ and $u =< 1, -2, 1 >$. Find the following:

   (a) $v \cdot u$,
   $v \cdot u = 9 \cdot 1 + 5(-2) + 1 \cdot 1 = 9 - 10 + 1 = 0.$

   (b) the angle between v and u.
   the angle is $90°$ *because the dot product of* v *and* u *is* 0.

2. Find an equation of the plane that passes through the point $(5, 4, 3)$ and is parallel to the plane $x - y + z = 0$.

   The vector $< 1, -1, 1 >$ *is normal to the plane* $x - y + z = 0$, *and thus is normal to the plane whose equation we want to find. Therefore an equation can be written as* $1(x - 5) + (-1)(y - 4) + 1(z - 3) = 0$, *or* $x - y + z - 4 = 0$.

3. Find equations of the line that passes through points $(0, 1, 2)$ and $(1, 2, 4)$.

   The vector from the first point to the second point is $< 1 - 0, 2 - 1, 4 - 2 >=< 1, 1, 2 >$. *This vector is parallel to the line. Using this vector and the first point, symmetric equations of the line can be written as* $\dfrac{x-0}{1} = \dfrac{y-1}{1} = \dfrac{z-2}{2}$, *or* $x = y - 1 = \dfrac{z-2}{2}$. *(Parametric equations are* $x = t,\ y = 1 + t,\ z = 2 + 2t$.*)*

4. Find and descibe the domain of $f(x, y, z) = \ln(1 - x^2 - y^2 - z^2)$.

   The function $\ln(1-x^2-y^2-z^2)$ *is defined whenever* $1-x^2-y^2-z^2 > 0$, *i.e.* $x^2+y^2+z^2 < 1$. *This inequality describes the interior of the sphere with center at the origin and radius* 1, *i.e. the open ball with center at the origin and radius* 1.

5. Consider the curve given by $r(t) =< t^2, t^3 + t^2, t^3 >$.

   (a) Find $r'(t)$.
   $r'(t) =< 2t, 3t^2 + 2t, 3t^2 >$.

   (b) Is this curve smooth? Explain why or why not.
   Since $r'(0) = 0$, *the curve is not smooth at* $t = 0$ *(but is smooth at all other points).*

6. (For extra credit) Find the point on the plane $2x + 3y + 4z + 5 = 0$ closest to the point $(1, 1, 1)$.

   The line joining $(1, 1, 1)$ *and the point on the plane* $2x+3y+4z+5 = 0$ *closest to* $(1, 1, 1)$ *is perpendicular to the given plane. Since the vector* $< 2, 3, 4 >$ *is normal to the plane, we can write equations of this line as* $x = 1 + 2t,\ y = 1 + 3t,\ z = 1 + 4t$. *Now let's find the intersection point of this line and the given plane. To do this, we'll substitute the above expressions for* x, y, *and* z *into the equation of the plane:* $2(1+2t)+3(1+3t)+4(1+4t)+5 = 0$. *Simplifying this gives* $29t + 14 = 0$. *Therefore* $t = -\dfrac{14}{29}$. *The intersection point thus has coordinates* $x = 1 + 2t = \dfrac{1}{29}$, $y = 1 + 3t = -\dfrac{13}{29}$, $z = 1 + 4t = -\dfrac{27}{29}$. *So the point on* $2x + 3y + 4z + 5 = 0$ *closest to* $(1, 1, 1)$ *is* $\left(\dfrac{1}{29}, -\dfrac{13}{29}, -\dfrac{27}{29}\right)$.