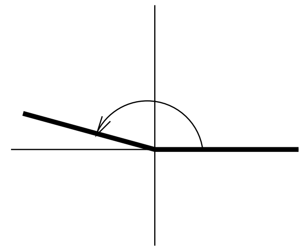
MATH 5

Test 1 – Solutions

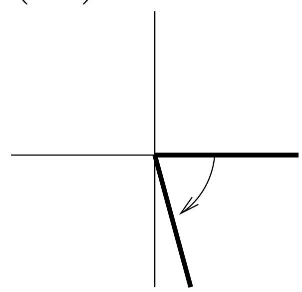
1. Convert 160° to radians and draw this angle in the standard position.

   Since $180^\circ = \pi$ rad, $1^\circ = \frac{\pi}{180}$ rad, so $160^\circ = \frac{160\pi}{180}$ rad $= \frac{8\pi}{9}$ rad.



2. Convert $-\frac{2\pi}{5}$ to degrees and draw this angle in the standard position.

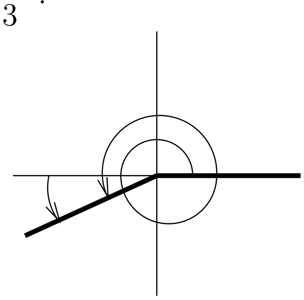
   Since $180^\circ = \pi$ rad, $1 \text{ rad} = \left(\frac{180}{\pi}\right)^\circ$, so $-\frac{2\pi}{5}$ rad $= -\frac{360\pi}{5\pi} = -72^\circ$.



3. Find an angle between 0° and 360° that is coterminal with the angle −250°.

   $-250^\circ + 360^\circ = 110^\circ$.

4. Find the reference angle of $\frac{10\pi}{3}$.



   Since $\frac{10\pi}{3} = 3\pi + \frac{\pi}{3}$, we see from the above picture that the reference angle is $\frac{\pi}{3}$.

5. If the terminal side of angle θ in the standard position passes through the point $(0.6, 0.8)$, find $\cos\theta$.

   The point $(0.6, 0.8)$ lies on the unit circle, so using the definition, $\cos\theta = 0.6$.

6. If the terminal side of angle θ in the standard position passes through the point $(0.6, 0.8)$, find $\tan\theta$.

   Using the definition, $\sin\theta = 0.8$, so $\tan\theta = \dfrac{0.8}{0.6} = \dfrac{4}{3}$.

7. If the terminal side of angle θ in the standard position passes through the point $(-6, 3)$, find $\sin\theta$.

   For $x = -6$ and $y = 3$, $r = \sqrt{x^2 + y^2} = \sqrt{36+9} = \sqrt{45} = 3\sqrt{5}$.
   Then $\sin\theta = \dfrac{y}{r} = \dfrac{3}{3\sqrt{5}} = \dfrac{1}{\sqrt{5}}$.

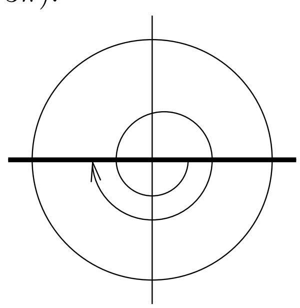
8. If θ is in quadrant III and $\sin\theta = -\dfrac{1}{7}$, find $\cos\theta$.

   Using $\cos^2\theta + \sin^2\theta = 1$, we have
   $\cos\theta = \pm\sqrt{1-\sin^2\theta} = \pm\sqrt{1-\dfrac{1}{49}} = \pm\sqrt{\dfrac{48}{49}} = \pm\dfrac{\sqrt{48}}{\sqrt{49}} = \pm\dfrac{4\sqrt{3}}{7}$.
   Since θ is in quadrant III, $\cos\theta$ is negative, so $\cos\theta = -\dfrac{4\sqrt{3}}{7}$.

9. If $\csc\theta = 4$, find $\sin\theta$.

   Using $\csc\theta = \dfrac{1}{\sin\theta}$, we have $\dfrac{1}{\sin\theta} = 4$, so $1 = 4\sin\theta$, therefore $\sin\theta = \dfrac{1}{4}$.

10. Find the exact value of $\cos(-3\pi)$.



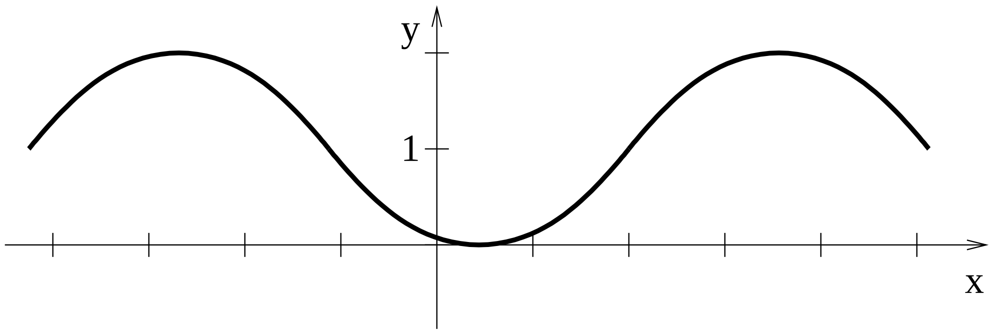
   Using the definition, $\cos(-3\pi) = -1$.

11. Find the exact value of $\tan\left(\dfrac{2\pi}{3}\right)$.

   The angle $\dfrac{2\pi}{3}$ is in quadrant II, so $\cos\left(\dfrac{2\pi}{3}\right)$ is negative and $\sin\left(\dfrac{2\pi}{3}\right)$ is positive. The reference angle is $\frac{\pi}{3}$, and from the triangle with sides 1 (adjacent side), $\sqrt{3}$ (opposite side), and 2 (hypotenuse) we know that $\cos\left(\dfrac{\pi}{3}\right) = \dfrac{1}{2}$ and $\sin\left(\dfrac{\pi}{3}\right) = \dfrac{\sqrt{3}}{2}$. So $\cos\left(\dfrac{2\pi}{3}\right) = -\dfrac{1}{2}$ and $\sin\left(\dfrac{2\pi}{3}\right) = \dfrac{\sqrt{3}}{2}$. Therefore $\tan\left(\dfrac{2\pi}{3}\right) = \dfrac{\sin\left(\frac{2\pi}{3}\right)}{\cos\left(\frac{2\pi}{3}\right)} = \dfrac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = -\sqrt{3}$.

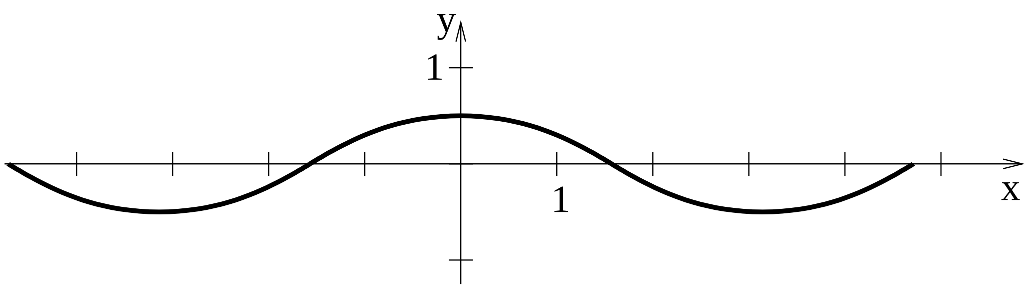
12. Sketch the graph of $\sin(x-2)+1$.

Shift the graph of $\sin x$ 2 units to the right and 1 unit upward:



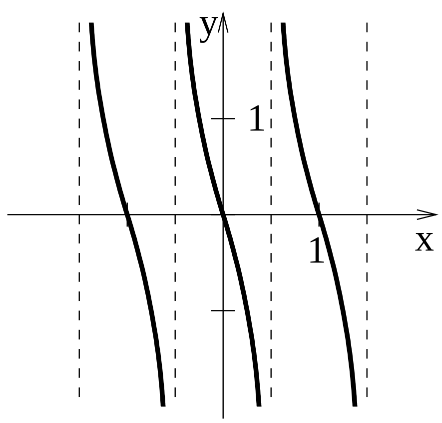
13. Sketch the graph of $0.5\cos(x+2\pi)$.

    Stretch the graph of $\cos x$ vertically by a factor of π (shifting by 2π units won't change it):

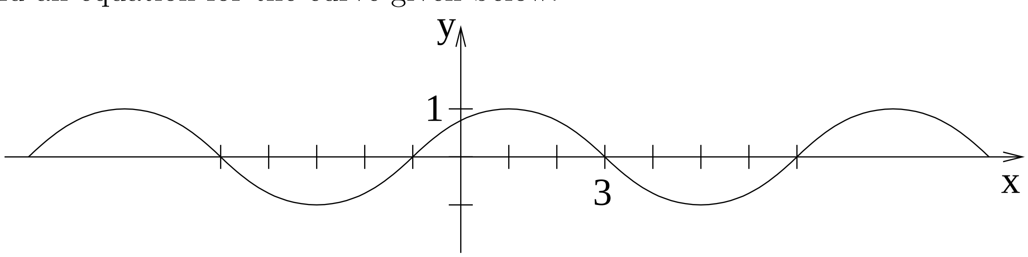


14. Sketch the graph of $-\tan(\pi x)$.

    Reflect the graph of $\tan x$ about the x-axis and compress horizontally by a factor of π:



15. Find an equation for the curve given below.



The given curve can be obtained from the curve $y = \sin x$ by stretching it horizontally (we have to determine by which factor) and then shifting 1 unit to the left. So the equation will have the form $y = \sin(b(x+1))$ where b can be found using the period. Namely, the period of such a function is $\dfrac{2\pi}{b}$, and the period of the given curve is 8. So we have $\dfrac{2\pi}{b} = 8$. Then $2\pi = 8b$, so $b = \dfrac{2\pi}{8} = \dfrac{\pi}{4}$. Thus one possible answer is $y = \sin\left(\dfrac{\pi}{4}(x+1)\right)$.

Note: there are other possible correct answers here since the curve may also be obtained using another shift or using the curve $y = \cos x$. Here are some other correct answers: $y = \sin\left(\dfrac{\pi}{4}(x-7)\right)$, $y = \cos\left(\dfrac{\pi}{4}(x-1)\right)$, $y = \cos\left(\dfrac{\pi}{4}(x+7)\right)$. There are some others, too. If your answer differs from all of the above, plot a few points and check whether they are on the given curve.