Math 75A **Maria Nogin**

Practice test 1 - Solutions

Multiple choice questions: circle the correct answer

1. The function $f(x) = \sin(x) + x^2$ is

A. even **B.** odd **C.** both even and odd (**D.**) neither even nor odd

2. If we shift the graph of $y = \sin(x)$ 2 units to the left, then the equation of the new graph is

A. $y = \sin(x) + 2$ **B.** $y = \sin(x) - 2$ (**C.**) $y = \sin(x + 2)$ **D.** $y = \sin(x - 2)$

E. $y = \sin(x/2)$

3. The domain of the function $f(x) = \dfrac{1}{\sqrt{x-1}}$ is the set of all real numbers x for which

A. $x > 0$ **B.** $x \neq 0$ **C.** $x \geq 1$ (**D.**) $x > 1$ **E.** $x \neq 1$

4. Simplify $\dfrac{1+x}{x} - \dfrac{\frac{1}{x}+1}{x+1}$.

(**A.**) 1 **B.** x **C.** $x + 1$ **D.** $\dfrac{1}{x}$ **E.** $\dfrac{x-1}{x+1}$

5. Let $f(x) = \begin{cases} -x-2 & \text{if } x < -1 \\ x-3 & \text{if } -1 \leq x \leq 1 \\ 2-x^2 & \text{if } x > 1 \end{cases}$. Find $f(1)$.

A. -3 (**B.**) -2 **C.** -1 **D.** 0 **E.** 1

6. If $f(x) = 1 + x$ and $g(x) = x^2 - 6$, find $(f \circ g)(-2)$.

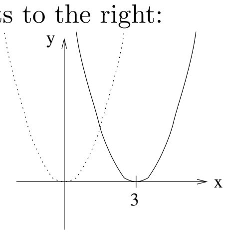
A. -9 **B.** -7 **C.** -5 (**D.**) -1 **E.** Undefined

Regular problems: show all your work

7. Use transformations of functions to sketch the graphs of:

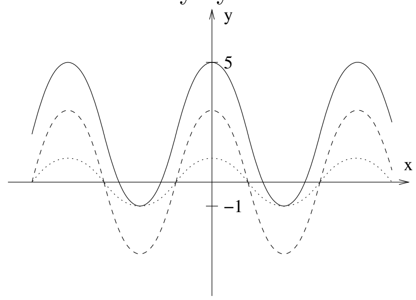
(a) $(x - 3)^2$

Shift the curve $y = x^2$ 3 units to the right:



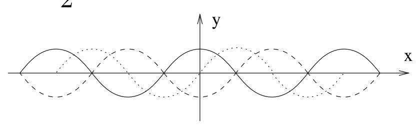
(b) $3\cos x + 2$

Stretch the curve $y = \cos x$ vertically by a factor of 3 and then shift 2 units upward:



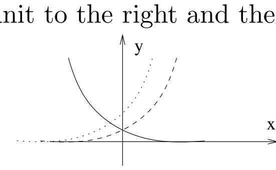
(c) $-\sin\left(x - \dfrac{\pi}{2}\right)$

Shift the curve $y = \sin x$ $\dfrac{\pi}{2}$ units to the right and then reflect about the x-axis:



(d) e^{-x-1}

Shift the curve $y = e^x$ 1 unit to the right and then reflect about the y-axis:



8. Find a formula for the function whose graph is obtained from the graph of $f(x) = e^x - 1$ by

(a) Reflecting about the y-axis and then compressing horizontally by a factor of 2.
Reflecting about the y-axis: $y = e^{-x} - 1$
Compressing horizontally by a factor of 2: $y = e^{-2x} - 1$

(b) Vertically compressing by a factor of 5 and then shifting 3 units to the left.
Vertically compressing by a factor of 5: $y = \dfrac{e^x - 1}{5}$
Shifting 3 units to the left: $y = \dfrac{e^{x+3} - 1}{5}$

(c) Reflecting about the x-axis and then shifting 2 units down.
Reflecting about the x-axis: $y = -(e^x - 1) = -e^x + 1$
Shifting 2 units down: $y = -e^x + 1 - 2 = -e^x - 1$

9. Let $f(x) = 2 - x$, $g(x) = \dfrac{1}{x}$, $h(x) = \sqrt{x+1}$. Find the following functions and their domains:

(a) $(f+g)(x) = 2 - x + \dfrac{1}{x}$
Domain $= (-\infty, 0) \cup (0, \infty)$

(b) $(f-g)(x) = 2 - x - \dfrac{1}{x}$
Domain $= (-\infty, 0) \cup (0, \infty)$

(c) $(fg)(x) = (2-x) \cdot \dfrac{1}{x} = \dfrac{2-x}{x}$
Domain $= (-\infty, 0) \cup (0, \infty)$

(d) $\left(\frac{f}{g}\right)(x) = \frac{2-x}{\frac{1}{x}} = 2x - x^2$ (if $x \neq 0$)
Domain $= (-\infty, 0) \cup (0, \infty)$

(e) $(g \circ f)(x) = \frac{1}{2-x}$
Domain $= (-\infty, 2) \cup (2, \infty)$

(f) $(f \circ h)(x) = 2 - \sqrt{x+1}$
Domain $= [-1, \infty)$

(g) $(g \circ h)(x) = \frac{1}{\sqrt{x+1}}$
Domain $= (-1, \infty)$

(h) $(f \circ g \circ h)(x) = 2 - \frac{1}{\sqrt{x+1}}$
Domain $= (-1, \infty)$

10. Find the inverse function of:

(a) $f(x) = 5x - 4$
$5x - 4 = y$
$5x = y + 4$
$x = \frac{y+4}{5}$
$f^{-1}(y) = \frac{y+4}{5}$
$f^{-1}(x) = \frac{x+4}{5}$

(b) $f(x) = (x+1)^3$
$(x+1)^3 = y$
$x + 1 = \sqrt[3]{y}$
$x = \sqrt[3]{y} - 1$
$f^{-1}(y) = \sqrt[3]{y} - 1$
$f^{-1}(x) = \sqrt[3]{x} - 1$

(c) $f(x) = e^x + 5$
$e^x + 5 = y$
$e^x = y - 5$
$x = \ln(y-5)$
$f^{-1}(y) = \ln(y-5)$
$f^{-1}(x) = \ln(x-5)$

11. Find the distance between $(-4, 3)$ and $(2, 11)$.
$D = \sqrt{(2-(-4))^2 + (11-3)^2} = \sqrt{6^2 + 8^2} = \sqrt{36+64} = \sqrt{100} = 10$

12. Write an equation of the circle

(a) whose radius is 3 and center is at $(3, -4)$
$(x-3)^2 + (y-(-4))^2 = 3^2$
$(x-3)^2 + (y+4)^2 = 9$

(b) whose center is at $(-2, 0)$ and that passes through the point $(1, 4)$
$r = \sqrt{(1-(-2))^2 + (4-0)^2} = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$
$(x-(-2))^2 + (y-0)^2 = 5^2$
$(x+2)^2 + y^2 = 25$

13. Write an equation of the line that

   (a) has slope 2 and passes through the point $(-1, 3)$
   $y - 3 = 2(x - (-1))$
   $y - 3 = 2(x + 1)$
   $y - 3 = 2x + 2$
   $y = 2x + 5$

   (b) passes throught the points $(-1, 3)$ and $(0, -6)$
   $m = \frac{-6 - 3}{0 - (-1)} = \frac{-9}{1} = -9$
   $y - 3 = -9(x - (-1))$
   $y - 3 = -9(x + 1)$
   $y - 3 = -9x - 9$
   $y = -9x - 6$

   (c) is parallel to the line $y = 7x - 1$ and passes through $(0, -6)$
   $m = 7$
   $b = -6$
   $y = 7x - 6$

   (d) is perpendicular to the line $y = 7x - 1$ and passes through $(0, -6)$
   $m = -\frac{1}{7}$
   $b = -6$
   $y = -\frac{1}{7}x - 6$

14. Evaluate the following expressions:

   (a) $\frac{2^5\sqrt{2^{20}}}{2^{18}}$
   $\frac{2^5\sqrt{2^{20}}}{2^{18}} = \frac{2^5 \cdot (2^{20})^{1/2}}{2^{18}} = \frac{2^5 \cdot 2^{10}}{2^{18}} = \frac{2^{15}}{2^{18}} = 2^{-3} = \frac{1}{8}$

   (b) $\log_2 32 = 5$ because $2^5 = 32$

   (c) $\log_4\left(\frac{1}{2}\right) = \log_4\left(\frac{1}{4^{1/2}}\right) = \log_4\left(4^{-1/2}\right) = -\frac{1}{2}$ because $\log_a a^x = x$ for all a and x

   (d) $3^{\log_3 7} = 7$ because $a^{\log_a x} = x$ for all a and x

   (e) $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

   (f) $\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

   (g) $\arcsin(1) = \frac{\pi}{2}$ because $\sin\left(\frac{\pi}{2}\right) = 1$

   (h) $\arccos\left(\frac{1}{2}\right) = \frac{\pi}{3}$ because $\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$