Math 75A **Maria Nogin**

Practice test 3 - Solutions

Multiple choice questions: circle the correct answer

1. Solve for x: $\log_{\frac{1}{2}} x = 3$.

   A. 6 **B.** $\frac{1}{6}$ **C.** 8 **(D.)** $\frac{1}{8}$
   E. None of the above

2. If $f(x) = x + \ln(x)$, find $f'(x)$.

   (A.) $\frac{x+1}{x}$ **B.** $\frac{1}{x}$ **C.** $1 - \frac{1}{x}$ **D.** $\frac{x}{x+1}$ **E.** $\frac{x}{x-1}$

3. If $f(x) = 4^{3x}$, find $f'(x)$.

   A. 4^{3x} **B.** $3 \cdot 4^{3x}$ **C.** 12^{3x} **D.** $\ln(4)4^{3x}$ **(E.)** $3\ln(4)4^{3x}$

4. Find the inverse function of $f(x) = x - 2$.

   A. $-x-2$ **B.** $-x+2$ **C.** $x-2$ **(D.)** $x+2$ **E.** $\frac{1}{x-2}$

5. Find the inverse function of $f(x) = 3^x$.

   A. -3^x **B.** $\frac{1}{3^x}$ **(C.)** $\log_3 x$ **D.** $\log_x 3$
   E. None of the above

6. Simplify: $\dfrac{\ln 8}{\ln 2}$

   (A.) 3 **B.** $\ln 3$ **C.** 4 **D.** $\ln 4$ **E.** $\ln 6$

7. What is the domain of the function $\ln x$?

   A. $\mathbb{R}$ **(B.)** $(0, +\infty)$ **C.** $[0, +\infty)$ **D.** $x \neq 0$
   E. None of the above

Regular problems: show all your work

8. Find the inverse function of:

   (a) $f(x) = 5x - 4$
   $y = 5x - 4$
   $y + 4 = 5x$
   $x = \frac{1}{5}(y+4)$
   $f^{-1}(y) = \frac{1}{5}(y+4)$
   $f^{-1}(x) = \frac{1}{5}(x+4)$

(b) $f(x) = (x+1)^3$
$y = (x+1)^3$
$\sqrt[3]{y} = x+1$
$x = \sqrt[3]{y} - 1$
$f^{-1}(y) = \sqrt[3]{y} - 1$
$f^{-1}(x) = \sqrt[3]{x} - 1$

(c) $f(x) = e^x + 5$
$y = e^x + 5$
$y - 5 = e^x$
$x = \ln(y-5)$
$f^{-1}(y) = \ln(y-5)$
$f^{-1}(x) = \ln(x-5)$

9. Find a formula for the function whose graph is obtained from the graph of $f(x) = e^x - 1$ by

(a) Reflecting about the y-axis and then compressing horizontally by a factor of 2.
Reflecting about the y-axis: $y = e^{-x} - 1$
Compressing horizontally by a factor of 2: $y = e^{-2x} - 1$

(b) Vertically compressing by a factor of 5 and then shifting 3 units to the left.
Vertically compressing by a factor of 5: $y = \dfrac{e^x - 1}{5}$
Shifting 3 units to the left: $y = \dfrac{e^{x+3} - 1}{5}$

(c) Reflecting about the x-axis and then shifting 2 units down.
Reflecting about the x-axis: $y = -(e^x - 1) = -e^x + 1$
Shifting 2 units down: $y = -e^x + 1 - 2 = -e^x - 1$

10. Evaluate the following expressions:

(a) $\dfrac{2^5\sqrt{2^{20}}}{2^{18}} = \dfrac{2^5 \cdot (2^{20})^{1/2}}{2^{18}} = \dfrac{2^5 \cdot 2^{10}}{2^{18}} = \dfrac{2^{15}}{2^{18}} = 2^{-3} = \dfrac{1}{8}$

(b) $\log_2 32 = 5$ because $2^5 = 32$

(c) $\log_5\left(\dfrac{1}{125}\right) = \log_5\left(\dfrac{1}{5^3}\right) = \log_5 5^{-3} = -3$ because $\log_a a^x = x$ for all a and x

(d) $\log_4\left(\dfrac{1}{2}\right) = \log_4\left(\dfrac{1}{4^{1/2}}\right) = \log_4\left(4^{-1/2}\right) = -\dfrac{1}{2}$

(e) $3^{\log_3 7} = 7$ because $a^{\log_a x} = x$ for all a and x

(f) $\log_6 2 + \log_6 3 = \log_6(2 \cdot 3) = \log_6 6 = 1$

(g) $3\log_8 4 = 3 \cdot \frac{\log_2 4}{\log_2 8} = 3 \cdot \frac{2}{3} = 2$

11. Solve the following equations:

(a) $\ln(5x-2) = 3$
$5x - 2 = e^3$
$5x = e^3 + 2$
$x = \frac{1}{5}(e^3 + 2)$

(b) $e^{3t+1} = 100$
$3t + 1 = \ln 100$
$3t = \ln 100 - 1$
$t = \frac{1}{3}(\ln 100 - 1)$

(c) $\log_2 t + \log_2(t+1) = 1$
$\log_2(t(t+1)) = 1$
$t(t+1) = 2$
$t^2 + t - 2 = 0$
$(t-2)(t+1) = 0$
$t = 2$ or $t = -1$
However, $\log_2 t$ is undefined at $t = -1$, so the only solution to the original equation is $t = 2$.

(d) $10^{4x+1} = 300$
$4x + 1 = \log_{10} 300$
$4x + 1 = \log_{10} 30 + \log_{10} 10$
$4x + 1 = \log_{10} 30 + 1$
$4x = \log_{10} 30$
$x = \frac{\log_{10} 30}{4}$

12. Differentiate the following functions:

(a) $f(x) = \left(\frac{1}{2}\right)^x$
$f'(x) = \left(\frac{1}{2}\right)^x \cdot \ln\left(\frac{1}{2}\right)$

(b) $f(x) = 5e^x - 8 \cdot 3^x + 9x^2$
$f'(x) = 5e^x - 8 \cdot 3^x \ln(3) + 18x$

(c) $f(x) = x^2 \ln x$
$f'(x) = 2x \ln x + x^2 \cdot \frac{1}{x} = 2x \ln x + x$

(d) $f(x) = \dfrac{\log_2 x - 2x}{2^x}$
$$f'(x) = \frac{\left(\frac{1}{x \ln 2} - 2\right) 2^x - (\log_2 x - 2x)2^x \ln 2}{(2^x)^2} = \frac{2^x \left(\left(\frac{1}{x \ln 2} - 2\right) - (\log_2 x - 2x) \ln 2\right)}{(2^x)^2} =$$
$$\frac{\left(\frac{1}{x \ln 2} - 2\right) - (\log_2 x - 2x) \ln 2}{2^x} = \frac{\frac{1}{x \ln 2} - 2 - \log_2 x \ln 2 + 2x \ln 2}{2^x} =$$
$$\frac{\frac{1}{x \ln 2} - 2 - \frac{\ln x}{\ln 2} \cdot \ln 2 + 2x \ln 2}{2^x} = \frac{\frac{1}{x \ln 2} - 2 - \ln x + 2x \ln 2}{2^x} =$$
$$\frac{1 - 2x \ln 2 - x \ln 2 \ln x + 2x^2 (\ln 2)^2}{2^x x \ln 2}$$

(e) $f(x) = \ln(x^3 + e^x)$
$$f'(x) = \frac{1}{x^3 + e^x} \cdot (3x^2 + e^x) = \frac{3x^2 + e^x}{x^3 + e^x}$$