

Practice test 1 - Solutions

Multiple choice questions: circle the correct answer

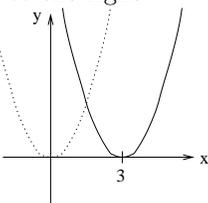
1. The function $f(x) = \sin(x) + x^2$ is
- A. even B. odd C. both even and odd **(D.)** neither even nor odd
2. If we shift the graph of $y = \sin(x)$ 2 units to the left, then the equation of the new graph is
- A. $y = \sin(x) + 2$ B. $y = \sin(x) - 2$ **(C.)** $y = \sin(x + 2)$ D. $y = \sin(x - 2)$
- E. $y = \sin(x/2)$
3. The domain of the function $f(x) = \frac{1}{\sqrt{x-1}}$ is the set of all real numbers x for which
- A. $x > 0$ B. $x \neq 0$ C. $x \geq 1$ **(D.)** $x > 1$ E. $x \neq 1$
4. Simplify $\frac{1+x}{x} - \frac{\frac{1}{x} + 1}{x+1}$.
- (A.)** 1 B. x C. $x + 1$ D. $\frac{1}{x}$ E. $\frac{x-1}{x+1}$
5. Let $f(x) = \begin{cases} -x-2 & \text{if } x < -1 \\ x-3 & \text{if } -1 \leq x \leq 1 \\ 2-x^2 & \text{if } x > 1 \end{cases}$. Find $f(1)$.
- A. -3 **(B.)** -2 C. -1 D. 0 E. 1
6. If $f(x) = 1 + x$ and $g(x) = x^2 - 6$, find $(f \circ g)(-2)$.
- A. -9 B. -7 C. -5 **(D.)** -1 E. Undefined

Regular problems: show all your work

7. Use transformations of functions to sketch the graphs of:

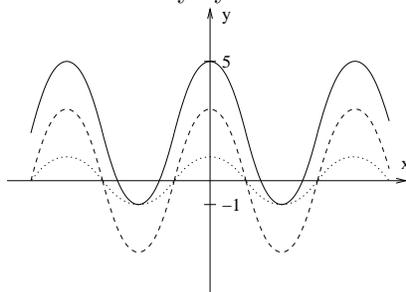
(a) $(x - 3)^2$

Shift the curve $y = x^2$ 3 units to the right:



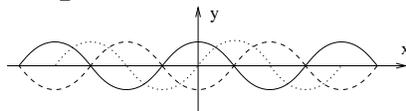
(b) $3 \cos x + 2$

Stretch the curve $y = \cos x$ vertically by a factor of 3 and then shift 2 units upward:



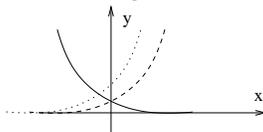
(c) $-\sin\left(x - \frac{\pi}{2}\right)$

Shift the curve $y = \sin x$ $\frac{\pi}{2}$ units to the right and then reflect about the x -axis:



(d) e^{-x-1}

Shift the curve $y = e^x$ 1 unit to the right and then reflect about the y -axis:



8. Find a formula for the function whose graph is obtained from the graph of $f(x) = e^x - 1$ by

(a) Reflecting about the y -axis and then compressing horizontally by a factor of 2.

Reflecting about the y -axis: $y = e^{-x} - 1$

Compressing horizontally by a factor of 2: $y = e^{-2x} - 1$

(b) Vertically compressing by a factor of 5 and then shifting 3 units to the left.

Vertically compressing by a factor of 5: $y = \frac{e^x - 1}{5}$

Shifting 3 units to the left: $y = \frac{e^{x+3} - 1}{5}$

(c) Reflecting about the x -axis and then shifting 2 units down.

Reflecting about the x -axis: $y = -(e^x - 1) = -e^x + 1$

Shifting 2 units down: $y = -e^x + 1 - 2 = -e^x - 1$

9. Let $f(x) = 2 - x$, $g(x) = \frac{1}{x}$, $h(x) = \sqrt{x + 1}$. Find the following functions and their domains:

(a) $(f + g)(x) = 2 - x + \frac{1}{x}$

Domain = $(-\infty, 0) \cup (0, \infty)$

(b) $(f - g)(x) = 2 - x - \frac{1}{x}$

Domain = $(-\infty, 0) \cup (0, \infty)$

(c) $(fg)(x) = (2 - x) \cdot \frac{1}{x} = \frac{2 - x}{x}$

Domain = $(-\infty, 0) \cup (0, \infty)$

(d) $\left(\frac{f}{g}\right)(x) = \frac{2 - x}{\frac{1}{x}} = 2x - x^2$ (if $x \neq 0$)

Domain = $(-\infty, 0) \cup (0, \infty)$

(e) $(g \circ f)(x) = \frac{1}{2 - x}$

Domain = $(-\infty, 2) \cup (2, \infty)$

(f) $(f \circ h)(x) = 2 - \sqrt{x + 1}$

Domain = $[-1, \infty)$

(g) $(g \circ h)(x) = \frac{1}{\sqrt{x + 1}}$

Domain = $(-1, \infty)$

(h) $(f \circ g \circ h)(x) = 2 - \frac{1}{\sqrt{x + 1}}$

Domain = $(-1, \infty)$

10. Find the distance between $(-4, 3)$ and $(2, 11)$.

$$D = \sqrt{(2 - (-4))^2 + (11 - 3)^2} = \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10$$

11. Write an equation of the circle

(a) whose radius is 3 and center is at $(3, -4)$

$$(x - 3)^2 + (y - (-4))^2 = 3^2$$

$$(x - 3)^2 + (y + 4)^2 = 9$$

(b) whose center is at $(-2, 0)$ and that passes through the point $(1, 4)$

$$r = \sqrt{(1 - (-2))^2 + (4 - 0)^2} = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$$

$$(x - (-2))^2 + (y - 0)^2 = 5^2$$

$$(x + 2)^2 + y^2 = 25$$

12. Write an equation of the line that

(a) has slope 2 and passes through the point $(-1, 3)$

$$y - 3 = 2(x - (-1))$$

$$y - 3 = 2(x + 1)$$

$$y - 3 = 2x + 2$$

$$y = 2x + 5$$

(b) passes through the points $(-1, 3)$ and $(0, -6)$

$$m = \frac{-6 - 3}{0 - (-1)} = \frac{-9}{1} = -9$$

$$y - 3 = -9(x - (-1))$$

$$y - 3 = -9(x + 1)$$

$$y - 3 = -9x - 9$$

$$y = -9x - 6$$

(c) is parallel to the line $y = 7x - 1$ and passes through $(0, -6)$

$$m = 7$$

$$b = -6$$

$$y = 7x - 6$$

(d) is perpendicular to the line $y = 7x - 1$ and passes through $(0, -6)$

$$m = -\frac{1}{7}$$

$$b = -6$$

$$y = -\frac{1}{7}x - 6$$

13. Evaluate the following expressions:

(a) $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

(b) $\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

(c) $\tan\left(-\frac{\pi}{3}\right) = \frac{\sin\left(-\frac{\pi}{3}\right)}{\cos\left(-\frac{\pi}{3}\right)} = \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = -\sqrt{3}$

(d) $\sec\left(\frac{3\pi}{4}\right) = \frac{1}{\cos\left(\frac{3\pi}{4}\right)} = \frac{1}{-\frac{\sqrt{2}}{2}} = -\frac{2}{\sqrt{2}} = -\sqrt{2}$