

## Practice test 1

The actual exam will consist of 6 multiple choice questions and 6 regular problems.  
You will have 1 hour to complete the exam.

## Multiple choice questions: circle the correct answer

- Find the exact value of  $\arcsin(1)$ .  
A. 0                      B.  $\frac{\pi}{2}$                       C.  $\pi$                       D.  $\frac{3\pi}{2}$                       E.  $2\pi$
- Find the exact value of  $\arccos\left(\frac{1}{2}\right)$ .  
A. 0                      B.  $\frac{\pi}{6}$                       C.  $\frac{\pi}{4}$                       D.  $\frac{\pi}{3}$                       E.  $\frac{\pi}{2}$
- Find the exact value of  $\sin\left(\arctan\left(\frac{3}{4}\right)\right)$ .  
A.  $-\frac{3}{5}$                       B.  $-\frac{3}{4}$                       C.  $\frac{3}{5}$                       D.  $\frac{3}{4}$                       E.  $\frac{4}{5}$
- Suppose 100 dollars are invested at an annual interest rate of 10% while interest is compounded monthly. What is the amount after 10 years?  
A.  $100\left(1 + \frac{1}{120}\right)^{10}$                       B.  $100\left(1 + \frac{1}{120}\right)^{120}$                       C.  $100\left(1 + \frac{10}{12}\right)^{10}$   
D.  $120\left(1 + \frac{10}{12}\right)^{100}$                       E.  $120\left(1 + \frac{1}{120}\right)^{100}$
- How many critical numbers does the function  $y = x + \frac{1}{x}$  have?  
A. 0                      B. 1                      C. 2                      D. 3                      E. infinitely many
- Find the local maximum of  $y = x + \frac{1}{x}$ .  
A.  $x = -2$                       B.  $x = -1$                       C.  $x = 0$                       D.  $x = 1$                       E.  $x = 2$

## Regular problems: show all your work

- Consider the curve given by  $x^3y^3 - 3xy^3 + 4y = 6$ .
  - Use implicit differentiation to find  $y'(x)$ .
  - Check that the point  $(2, 1)$  lies on this curve.
  - What is the slope of the tangent line to this curve at  $(2, 1)$ ?
- Find the slope of the tangent line to the curve  $x \tan y + xy + 3y = 0$  at the point  $(0, 0)$ .

9. A boy starts walking west at 6 km/h from a point  $P$ . Five minutes later a girl starts walking
- north
  - east
- at 4 km/h from a point 15 km due south from  $P$ . At what rate is the distance between the kids changing 45 min after the girl starts walking? Is the distance increasing or decreasing at this instant?
10. A snowball is melting so that its radius is decreasing at a rate of 1 cm/min. Find the rate at which its volume is decreasing when the radius is 3 cm.
11. A bacteria culture starts with 800 bacteria and the growth rate is proportional to the number of bacteria. After 3 hours the population is 2700. Find the number of bacteria after 5 hours.
12. Differentiate the following functions.
- $f(x) = \arcsin(3x)$
  - $g(x) = x \tan^{-1}(1 - x)$
  - $h(x) = \frac{\arccos(x)}{\sqrt{1-x^2}}$
13. Evaluate the following limits.
- $\lim_{x \rightarrow 0} \frac{\sin 5x}{2 \sin 3x}$
  - $\lim_{x \rightarrow 0} \frac{e^x(\cos x - 1)}{\tan(3x)}$
  - $\lim_{x \rightarrow 0} \frac{e^x - 1 - x}{x^2}$
  - $\lim_{x \rightarrow \infty} x^3 e^{-3x}$
  - $\lim_{x \rightarrow \infty} \left( \frac{x}{x+1} \right)^{3x}$
14. Find the critical numbers and local maxima and minima of  $f(x) = x^3 - 3x^2 + 5$ .
15. Find the absolute maximum and minimum values of  $f(x) = x^4 + 4x^3 + 5$  on the interval  $[-2, 0]$ .
16. Find the absolute maximum and minimum values of  $f(x) = \sin x$  on the interval  $\left[0, \frac{5\pi}{4}\right]$ .
17. Show that the equation  $x^7 + 3x^3 + x = 4$  has exactly one real root.