Math 75B **Maria Nogin**

Practice test 1 - Solutions

Multiple choice questions: circle the correct answer

1. Find the exact value of $\arcsin(1)$.

   A. 0 (**B.**) $\frac{\pi}{2}$ **C.** π **D.** $\frac{3\pi}{2}$ **E.** 2π

2. Find the exact value of $\arccos\left(\frac{1}{2}\right)$.

   A. 0 **B.** $\frac{\pi}{6}$ **C.** $\frac{\pi}{4}$ (**D.**) $\frac{\pi}{3}$ **E.** $\frac{\pi}{2}$

3. Find the exact value of $\sin\left(\arctan\left(\frac{3}{4}\right)\right)$.

   A. $-\frac{3}{5}$ **B.** $-\frac{3}{4}$ (**C.**) $\frac{3}{5}$ **D.** $\frac{3}{4}$ **E.** $\frac{4}{5}$

4. Suppose 100 dollars are invested at an annual intererst rate of 10% while interest is compounded monthly. What is the ammount after 10 years?

   A. $100\left(1+\frac{1}{120}\right)^{10}$ (**B.**) $100\left(1+\frac{1}{120}\right)^{120}$ **C.** $100\left(1+\frac{10}{12}\right)^{10}$
   D. $120\left(1+\frac{10}{12}\right)^{100}$ **E.** $120\left(1+\frac{1}{120}\right)^{100}$

5. How many critical numbers does the function $y = x + \frac{1}{x}$ have?

   A. 0 **B.** 1 (**C.**) 2 **D.** 3 **E.** infinitely many

6. Find the local maximum of $y = x + \frac{1}{x}$.

   A. $x = -2$ (**B.**) $x = -1$ **C.** $x = 0$ **D.** $x = 1$ **E.** $x = 2$

Regular problems: show all your work

7. (a) $3x^2y^3 + 3x^3y^2y' - 3y^3 - 9xy^2y' + 4y' = 0$

   $(3x^3y^2 - 9xy^2 + 4)y' = 3y^3 - 3x^2y^3$

   $$y' = \frac{3y^3 - 3x^2y^3}{3x^3y^2 - 9xy^2 + 4}$$

   (b) $2^3 - 3 \cdot 2 + 4 = 6$

   (c) $y'(2) = \dfrac{3 - 3 \cdot 2^2}{3 \cdot 2^3 - 9 \cdot 2 + 4} = -\dfrac{9}{10}$

8. $\tan y + x \sec^2 y \cdot y' + y + xy' + 3y' = 0$

$(x \sec^2 y + x + 3)y' = -\tan y - y$

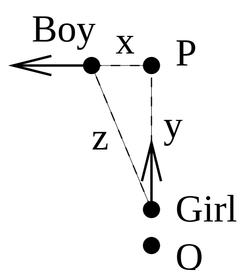
If $x = 0$ and $y = 0$, then $3y'(0) = 0$, so the slope of the tangent line is 0.

9. (a) Let x be the distance between the boy and the point P, let y be the distance between the girl and P, and let z be the distance between the boy and the girl. Then $x^2 + y^2 = z^2$ where x, y, and z are functions of time.

Differentiating this equation with respect to t gives

$2xx' + 2yy' = 2zz'$

$xx' + yy' = zz'$



45 minutes after the girl started walking (and thus 50 minutes after the boy started walking), $x = 6 \cdot \frac{50}{60} = 5$, $y = 15 - 4 \cdot 4560 = 15 - 3 = 12$, and $z = \sqrt{5^2 + 12^2} = 13$. x' is the rate of change of x, i.e. the speed of the boy, so $x' = 6$, and y' is the rate of change of y, i.e. negative the speed of the girl since y is decreasing, so $y' = -4$. Therefore

$5 \cdot 6 + 12 \cdot (-4) = 13z$

Answer: $-\dfrac{18}{13}$, decreasing.

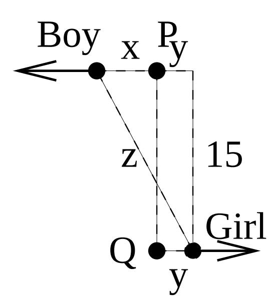
(b) Let x be the distance between the boy and the point P, let y be the distance between the girl and her starting point Q, and let z be the distance between the boy and the girl.

Then $(x + y)^2 + 15^2 = z^2$ (see the figure)

Differentiating this equation with respect to t gives

$2(x + y)(x' + y') = 2zz'$

$(x + y)(x' + y') = zz'$



45 minutes after the girl started walking (and thus 50 minutes after the boy started walking), $x = 6 \cdot \frac{50}{60} = 5$, $y = 4 \cdot 4560 = 3$, so $x + y = 8$, and $z = \sqrt{8^2 + 15^2} = 17$. x' is the rate of change of x, i.e. the speed of the boy, so $x' = 6$, and y' is the rate of change of y, i.e. the speed of the girl, so $y' = 4$. Therefore

$(5 + 3)(6 + 4) = 17z'$

Answer: $\dfrac{80}{17}$, increasing.

10. $V(t) = \frac{4}{3}\pi(r(t))^3$

    $V'(t) = 4\pi(r(t))^2 r'(t)$

    If $r' = -1$ and $r = 3$, $V'(t) = 4\pi 3^2 \cdot 1 = 36\pi$

    Answer: 36π cm^3/min.

11. Since initially there are 800 bacteria, $P(t) = 800e^{kt}$. At $t = 3$ we have: $2700 = 800e^{k\cdot 3}$ $(e^k)^3 = \frac{27}{8}$ $e^k = \frac{3}{2}$. Then at $t = 5$: $P(5) = 800e^{k\cdot 5} = 800(e^k)^5 = 800\left(\frac{3}{2}\right)^5 = \frac{800\cdot 3^5}{2^5} = 25 \cdot 343 = 6075$.

12. (a) $f'(x) = \frac{3}{\sqrt{1-9x^2}})$

    (b) $'g(x) = \tan^{-1}(1-x) + \frac{-x}{1+(1-x)^2}$

    (c) $h(x) = \frac{-\frac{\sqrt{1-x^2}}{\sqrt{1-x^2}} - \arccos(x)\frac{-2x}{2\sqrt{1-x^2}}}{1-x^2} = \frac{-1+\frac{x\arccos(x)}{\sqrt{1-x^2}}}{1-x^2}$

13. (a) $\lim_{x\to 0} \frac{\sin 5x}{2\sin 3x} = \lim_{x\to 0} \frac{5\cos 5x}{6\cos 3x} = \frac{5}{6}$

    (b) $\lim_{x\to 0} \frac{e^x(\cos x - 1)}{\tan(3x)} = 1 \cdot \lim_{x\to 0} \frac{\cos x - 1}{\tan(3x)} = \lim_{x\to 0} \frac{-\sin x}{3\sec^2(3x)} = 0$

    (c) $\lim_{x\to 0} \frac{e^x - 1 - x}{x^2} = \lim_{x\to 0} \frac{e^x - 1}{2x} = \lim_{x\to 0} \frac{e^x}{2} = \frac{1}{2}$

    (d) $\lim_{x\to\infty} x^3 e^{-3x} = \lim_{x\to\infty} \frac{x^3}{e^{3x}} = \lim_{x\to\infty} \frac{3x^2}{3e^{3x}} = \lim_{x\to\infty} \frac{x^2}{e^{3x}} = \lim_{x\to\infty} \frac{2x}{3e^{3x}} = \lim_{x\to\infty} \frac{2}{9e^{3x}} = 0$

    (e) $\lim_{x\to\infty} \left(\frac{x}{x+1}\right)^{3x} = \lim_{x\to\infty} \left(e^{\ln\frac{x}{x+1}}\right)^{3x} = e^{\lim_{x\to\infty} \ln\frac{x}{x+1} \cdot 3x} = e^{\lim_{x\to\infty} \frac{3\ln\frac{x}{x+1}}{1/x}} =$
    $e^{\lim_{x\to\infty} \frac{3\frac{x+1}{x} \cdot \frac{1}{(x+1)^2}}{-\frac{1}{x^2}}} = e^{\lim_{x\to\infty} \frac{\frac{3}{x(x+1)}}{-\frac{1}{x^2}}} = e^{\lim_{x\to\infty} -\frac{3x^2}{x(x+1)}} = e^{\lim_{x\to\infty} -\frac{3x}{x+1}} = e^{-3}$

14. $f'(x) = 3x^2 - 6x = 0$ gives $x = 0$ and $x = 2$. Since $f'(-1) > 0$, $f'(1) < 0$, and $f'(3) > 0$, the point $x = 0$ is a local minimum and the point $x = 2$ is a local maximum.

15. Use the closed interval method:
    1. Find the critical numbers: $f'(x) = 4x^3 + 12x^2 = 0$ gives $x = 0$ and $x = -3$. However, -3 is not in our interval. Only 0 is.
    2. Find the value of the function at the critical number(s): $f(0) = 5$.
    3. Find the value of the function at the endpoints of the interval: $f(-2) = -11$. The other endpoint is 0, but we already found the value at 0.
    4. The largest of the above values, i.e. 5, is the absolute maximum value of the function on the given interval, and the smallest of the above values, i.e. -11, is the absolute minimum value.

16. Use the closed interval method:
 1. Find the critical numbers: $f'(x) = \cos x$. The only root on the given interval is $x = \frac{\pi}{2}$.
 2. Find the value of the function at the critical number(s): $f\left(\frac{\pi}{2}\right) = 1$.
 3. Find the value of the function at the endpoints of the interval: $f(0) = 0$, $f\left(\frac{5\pi}{4}\right) = -\frac{1}{\sqrt{2}}$.
 4. The largest of the above values, i.e. 1, is the absolute maximum value of the function on the given interval, and the smallest of the above values, i.e. $-\frac{1}{\sqrt{2}}$, is the absolute minimum value.

17. Let $f(x) = x^7 + 3x^3 + x$. Since the funtion $f(x)$ is continuous, $f(0) < 4$ and $f(1) > 4$, by the Intermediate Value Theorem the equation $f(x) = 4$ has at least one real root. However, since $f'(x) = 7x^6 + 9x^2 + 1 > 0$, by Rolle's Theorem this equation cannot have more than one real root. Thus it has exactly one real root.