

MATH 75B

Final Exam - Version 1

December 16, 2009

Name: _____

- Please turn off your cell phones and any other electronic devices.
- Write your name both on the exam and on the scantron form. Also please write the exam version in the field “TEST NO.”
- No notes, books, or calculators are allowed.
- You may write on the exam. If you need additional scratch paper, please raise your hand.
- You have to turn in your scantron form, exam, and all your scratch paper.
- Show your photo ID to the instructor when you turn in your papers.

1. Evaluate $\int \frac{e^x}{1 + e^{2x}} dx$.

- (a) $\arctan(e^x) + C$ **correct**
 - (b) $\frac{1}{e^x + \frac{e^{3x}}{3}} + C$
 - (c) $\frac{e^x}{x + e^{2x}} + C$
 - (d) $\frac{e^x}{1 + e^{x^2}} + C$
 - (e) none of the above
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2. Evaluate $\int_0^1 \frac{dx}{\cos^2 x + \sin^2 x}$.

- (a) 0
 - (b) 1 **correct**
 - (c) $\frac{\pi}{2}$
 - (d) π
 - (e) 2π
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3. Bismuth-210 has a half-life of 5 days. A sample originally has a mass of 800 mg. Find the mass remaining after 30 days.

- (a) $\frac{e^{80/3}}{5}$
 - (b) 25
 - (c) $\frac{400}{3}$
 - (d) $\frac{800}{e^6}$
 - (e) none of the above **correct**
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4. On the interval $(-1, 1)$ the function $f(x) = \frac{1}{2}e^x - x - 1$ is

- (a) constant
- (b) positive
- (c) increasing
- (d) concave upward **correct**
- (e) even

5. Find the inflection points of $f(x) = 3x^5 + 10x^4 + 10x^3 + 3x + 10$.

- (a) $x = -1$
 - (b) $x = 0$ **correct**
 - (c) $x = 1$
 - (d) $x = -1$ and $x = 0$
 - (e) $x = 0$ and $x = 1$
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6. Evaluate $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^{x^2}$.

- (a) 0
 - (b) 1
 - (c) e
 - (d) π
 - (e) ∞ **correct**
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7. Let a rectangle be inscribed in the region between the x -axis and the parabola $y = 2 - x^2$ so that its base lies on the x -axis and its upper vertices lie on the parabola. Find the largest possible area of such a rectangle.

- (a) 2
 - (b) $2\sqrt{2}$
 - (c) $\frac{3\sqrt{3}}{2\sqrt{2}}$
 - (d) $\frac{8\sqrt{2}}{3\sqrt{3}}$ **correct**
 - (e) none of the above
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8. Which of the following would be the best approximation x_1 if we want to use Newton's method to estimate the root of $\cos x = 2x$?

- (a) $x = -\pi$
- (b) $x = \frac{1}{2}$ **correct**
- (c) $x = \frac{\pi}{2}$
- (d) $x = 2$
- (e) $x = \frac{3\pi}{2}$

9. Suppose $f'(x) = (x - 1)^2(x - 3)^5(x - 6)^7$. On what interval(s) is $f(x)$ decreasing?

- (a) $(-\infty, 1)$
 - (b) $(1, 3)$
 - (c) $(3, 6)$ **correct**
 - (d) $(-\infty, 1) \cup (3, 6)$
 - (e) $(1, 3) \cup (6, +\infty)$
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10. At which point(s) on the curve $y = 1 + 40x^3 - 3x^5$ does the tangent line have the largest slope?

- (a) $(0, 1)$
 - (b) $(\sqrt{2}, 1 + 68\sqrt{2})$ and $(-\sqrt{2}, 1 - 68\sqrt{2})$ **correct**
 - (c) $(1, 38)$ and $(-1, -36)$
 - (d) $(2, 225)$ and $(-2, -223)$
 - (e) none of the above
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11. Find the absolute maximum value of $f(x) = xe^{-x}$ on the interval $[-10, 10]$.

- (a) $-\frac{10}{e^{10}}$
 - (b) $10e^{-10}$
 - (c) $10e^{10}$
 - (d) 10
 - (e) none of the above **correct**
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12. Find an equation of the tangent line to the curve given by $x^2 + 2xy - y^2 + x = 2$ at the point $(1, 2)$.

- (a) $y = \frac{7}{2}x - \frac{3}{2}$ **correct**
 - (b) $y = \frac{5}{2}x - \frac{1}{2}$
 - (c) $y = -\frac{3}{2}x - \frac{7}{2}$
 - (d) $y = -\frac{1}{2}x + \frac{5}{2}$
 - (e) none of the above
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13. A spotlight on the ground shines on a wall 12 m away. If a man 2 m tall walks from the spotlight toward the building at a speed of 1.6 m/s, how fast is the length of his shadow on the building decreasing when he is 4 m from the building?

- (a) 0.4 m/s
- (b) 0.6 m/s **correct**
- (c) 0.8 m/s
- (d) 1.2 m/s
- (e) none of the above

14. Find the horizontal asymptotes of $f(x) = \arctan x$, if it has any.

- (a) $y = \frac{\pi}{2}$
- (b) $y = 0$
- (c) $y = \frac{\pi}{2}$ and $y = -\frac{\pi}{2}$ **correct**
- (d) $y = \pi$ and $y = -\pi$
- (e) no horizontal asymptotes

15. How many critical numbers does $f(x) = e^x \sin x$ have on the interval $[-5, 5]$?

- (a) 0
- (b) 1
- (c) 2
- (d) 3 **correct**
- (e) infinitely many

16. Evaluate $\arcsin\left(\sin\left(\frac{29}{20}\pi\right)\right)$.

- (a) $-\frac{11}{20}\pi$
- (b) $-\frac{9}{20}\pi$ **correct**
- (c) $-\frac{1}{10}\pi$
- (d) $\frac{1}{20}\pi$
- (e) $\frac{9}{10}\pi$

17. A car was traveling at 50 ft/s when the brakes are fully applied, producing a constant deceleration. If the car traveled 250 ft before coming to a stop, how long did it take it to stop?

- (a) 5 s
- (b) 7.5 s
- (c) 8.75 s
- (d) 10 s **correct**
- (e) 12.5 s

18. Evaluate $\lim_{x \rightarrow 5} \frac{\sqrt{x+4} - 3}{x-5}$

- (a) 0
- (b) 1
- (c) $\frac{1}{3}$
- (d) $\frac{1}{6}$ **correct**
- (e) ∞

19. Find the number c that satisfies the conclusion of the Mean Value Theorem for the function $f(x) = \frac{x}{x+2}$ and the interval $[1, 4]$.

- (a) $\frac{5}{9}$
- (b) -2
- (c) $\frac{1}{2}$
- (d) $\frac{5}{3}$
- (e) none of the above **correct**

20. Find the vertical asymptotes of the graph of $f(x) = \frac{x^2 + x}{x^2 + 3x + 2}$.

- (a) $x = 1$
- (b) $x = -1$
- (c) $x = -2$ **correct**
- (d) $x = -1$ and $x = -2$
- (e) $x = 1$ and $x = -1$

21. Estimate the area of the region bounded by $y = \frac{12}{x}$, $x = 1$, $x = 9$, and $y = 0$, using 4 approximating rectangles and midpoints.

- (a) 20
- (b) 25 **correct**
- (c) 30
- (d) 36
- (e) none of the above

22. Find the domain of the function $f(x) = \sqrt{\frac{1-x}{1+x}}$.

- (a) $(1, \infty)$
 - (b) $[1, \infty)$
 - (c) $(-1, 1]$ **correct**
 - (d) $(-\infty, -1)$
 - (e) $(-\infty, -1) \cup (-1, \infty)$.
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23. Let $f(x) = \int_1^x \arcsin\left(\frac{\sqrt{t^2+1}}{2}\right) dt$, find $f'(1)$.

- (a) 0
 - (b) $\frac{1}{2}$
 - (c) $\frac{\pi}{2}$
 - (d) $\frac{\pi}{4}$ **correct**
 - (e) none of the above
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24. A kite 100 ft above the ground moves horizontally at a speed of 8 ft/s. At what rate is the angle between the string and the ground decreasing when 200 ft of string has been let out?

- (a) 0.02 rad/s **correct**
 - (b) 0.04 rad/s
 - (c) 0.06 rad/s
 - (d) 0.08 rad/s
 - (e) none of the above
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25. A particle moves along a straight line with equation of motion $s(t) = \sqrt{t+1}$. Find its average velocity over the time interval $[0, 8]$.

- (a) $\frac{1}{4}$ **correct**
- (b) $\frac{1}{6}$
- (c) $\frac{1}{2}$
- (d) 0
- (e) 2