MATH 75B

Final Exam - Version 1

December 16, 2009

Name:_____

- Please turn off your cell phones and any other electronic devices.
- Write your name both on the exam and on the scantron form. Also please write the exam version in the field "TEST NO."
- No notes, books, or calculators are allowed.
- You may write on the exam. If you need additional scratch paper, please raise your hand.
- You have to turn in your scantron form, exam, and all your scratch paper.
- Show your photo ID to the instructor when you turn in your papers.

1. Evaluate $\displaystyle\int \frac{e^x}{1+e^{2x}}dx$.

   (a) $\arctan(e^x)+C$

   (b) $\frac{1}{e^x+\frac{e^{3x}}{3}}+C$

   (c) $\frac{e^x}{x+e^{2x}}+C$

   (d) $\frac{e^x}{1+e^{x^2}}+C$

   (e) none of the above

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2. Evaluate $\displaystyle\int_0^1 \frac{dx}{\cos^2 x+\sin^2 x}$.

   (a) 0

   (b) 1

   (c) $\frac{\pi}{2}$

   (d) π

   (e) 2π

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3. Bismuth-210 has a half-life of 5 days. A sample originally has a mass of 800 mg. Find the mass remaining after 30 days.

   (a) $\frac{e^{80/3}}{5}$

   (b) 25

   (c) $\frac{400}{3}$

   (d) $\frac{800}{e^6}$

   (e) none of the above

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4. On the interval $(-1,1)$ the function $f(x)=\frac{1}{2}e^x-x-1$ is

   (a) constant

   (b) positive

   (c) increasing

   (d) concave upward

   (e) even

5. Find the inflection points of $f(x) = 3x^5 + 10x^4 + 10x^3 + 3x + 10$.

   (a) $x = -1$
   (b) $x = 0$
   (c) $x = 1$
   (d) $x = -1$ and $x = 0$
   (e) $x = 0$ and $x = 1$

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6. Evaluate $\lim_{x\to\infty} \left(1 + \frac{1}{x}\right)^{x^2}$.

   (a) 0
   (b) 1
   (c) e
   (d) π
   (e) ∞

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7. Let a rectangle be inscirbed in the region between the x-axis and the parabola $y = 2 - x^2$ so that its base lies on the x-axis and its upper vertices lie on the parabola. Find the largest possible area of such a rectangle.

   (a) 2
   (b) $2\sqrt{2}$
   (c) $\frac{3\sqrt{3}}{2\sqrt{2}}$
   (d) $\frac{8\sqrt{2}}{3\sqrt{3}}$
   (e) none of the above

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8. Which of the following would be the best approximation x_1 if we want to use Newton's method to estimate the root of $\cos x = 2x$?

   (a) $x = -\pi$
   (b) $x = \frac{1}{2}$
   (c) $x = \frac{\pi}{2}$
   (d) $x = 2$
   (e) $x = \frac{3\pi}{2}$

9. Suppose $f'(x) = (x-1)^2(x-3)^5(x-6)^7$. On what interval(s) is $f(x)$ decreasing?

   (a) $(-\infty, 1)$
   (b) $(1, 3)$
   (c) $(3, 6)$
   (d) $(-\infty, 1) \cup (3, 6)$
   (e) $(1, 3) \cup (6, +\infty)$

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10. At which point(s) on the curve $y = 1 + 40x^3 - 3x^5$ does the tangent line have the largest slope?

    (a) $(0, 1)$
    (b) $(\sqrt{2}, 1 + 68\sqrt{2})$ and $(-\sqrt{2}, 1 - 68\sqrt{2})$
    (c) $(1, 38)$ and $(-1, -36)$
    (d) $(2, 225)$ and $(-2, -223)$
    (e) none of the above

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11. Find the absolute maximum value of $f(x) = xe^{-x}$ on the interval $[-10, 10]$.

    (a) $-\frac{10}{e^{10}}$
    (b) $10e^{-10}$
    (c) $10e^{10}$
    (d) 10
    (e) none of the above

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12. Find an equation of the tangent line to the curve given by $x^2 + 2xy - y^2 + x = 2$ at the point $(1, 2)$.

    (a) $y = \frac{7}{2}x - \frac{3}{2}$
    (b) $y = \frac{5}{2}x - \frac{1}{2}$
    (c) $y = -\frac{3}{2}x - \frac{7}{2}$
    (d) $y = -\frac{1}{2}x + \frac{5}{2}$
    (e) none of the above

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13. A spotlight on the ground shines on a wall 12 m away. If a man 2 m tall walks from the spotlight toward the building at a speed of 1.6 m/s, how fast is the length of his shadow on the building decreasing when he is 4 m from the building?

    (a) 0.4 m/s
    (b) 0.6 m/s
    (c) 0.8 m/s
    (d) 1.2 m/s
    (e) none of the above

14. Find the horizontal asymptotes of $f(x) = \arctan x$, if it has any.

(a) $y = \frac{\pi}{2}$

(b) $y = 0$

(c) $y = \frac{\pi}{2}$ and $y = -\frac{\pi}{2}$

(d) $y = \pi$ and $y = -\pi$

(e) no horizontal asymptotes

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15. How many critical numbers does $f(x) = e^x \sin x$ have on the interval $[-5, 5]$?

(a) 0

(b) 1

(c) 2

(d) 3

(e) infinitely many

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16. Evaluate $\arcsin\left(\sin\left(\frac{29}{20}\pi\right)\right)$.

(a) $-\frac{11}{20}\pi$

(b) $-\frac{9}{20}\pi$

(c) $-\frac{1}{10}\pi$

(d) $\frac{1}{20}\pi$

(e) $\frac{9}{10}\pi$

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17. A car was traveling at 50 ft/s when the brakes are fully applied, producing a constant deceleration. If the car traveled 250 ft before coming to a stop, how long did it take it to stop?

(a) 5 s

(b) 7.5 s

(c) 8.75 s

(d) 10 s

(e) 12.5 s

18. Evaluate $\lim_{x \to 5} \dfrac{\sqrt{x+4} - 3}{x - 5}$

(a) 0

(b) 1

(c) $\frac{1}{3}$

(d) $\frac{1}{6}$

(e) ∞

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19. Find the number c that satisfies the conclusion of the Mean Value Theorem for the function $f(x) = \frac{x}{x+2}$ and the interval $[1, 4]$.

(a) $\frac{5}{9}$

(b) -2

(c) $\frac{1}{2}$

(d) $\frac{5}{3}$

(e) none of the above

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20. Find the vertical asymptotes of the graph of $f(x) = \dfrac{x^2 + x}{x^2 + 3x + 2}$.

(a) $x = 1$

(b) $x = -1$

(c) $x = -2$

(d) $x = -1$ and $x = -2$

(e) $x = 1$ and $x = -1$

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21. Estimate the area of the region bounded by $y = \frac{12}{x}$, $x = 1$, $x = 9$, and $y = 0$, using 4 approximating rectangles and midpoints.

(a) 20

(b) 25

(c) 30

(d) 36

(e) none of the above

22. Find the domain of the function $f(x) = \sqrt{\dfrac{1-x}{1+x}}$.

(a) $(1, \infty)$

(b) $[1, \infty)$

(c) $(-1, 1]$

(d) $(-\infty, -1)$

(e) $(-\infty, -1) \cup (-1, \infty)$.

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23. Let $f(x) = \displaystyle\int_1^x \arcsin\left(\frac{\sqrt{t^2+1}}{2}\right) dt$, find $f'(1)$.

(a) 0

(b) $\frac{1}{2}$

(c) $\frac{\pi}{2}$

(d) $\frac{\pi}{4}$

(e) none of the above

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24. A kite 100 ft above the ground moves horizontally at a speed of 8 ft/s. At what rate is the angle between the string and the ground decreasing when 200 ft of string has been let out?

(a) 0.02 rad/s

(b) 0.04 rad/s

(c) 0.06 rad/s

(d) 0.08 rad/s

(e) none of the above

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25. A particle moves along a straight line with equation of motion $s(t) = \sqrt{t+1}$. Find its average velocity over the time interval $[0, 8]$.

(a) $\frac{1}{4}$

(b) $\frac{1}{6}$

(c) $\frac{1}{2}$

(d) 0

(e) 2