

Practice test 1

The actual exam will consist of 6 multiple choice questions and 6 regular problems.
You will have 1 hour to complete the exam.

Multiple choice questions: circle the correct answer

1. The function $f(x) = \sin(x) + x^2$ is
 - A. even
 - B. odd
 - C. periodic with period 2π
 - D. discontinuous at 0
 - E. None of the above

2. If we shift the graph of $y = \sin(x)$ 2 units to the left, then the equation of the new graph is
 - A. $y = \sin(x) + 2$
 - B. $y = \sin(x) - 2$
 - C. $y = \sin(x + 2)$
 - D. $y = \sin(x - 2)$
 - E. $y = \sin(x/2)$

3. The domain of the function $f(x) = \sqrt{\frac{1}{9-x^2}} + \sqrt{x-1}$ is the set of all real numbers x for which
 - A. $x < -3$ or $x > 3$
 - B. $x < 3$
 - C. $x \geq 1$
 - D. $1 \leq x < 3$
 - E. $1 < x < 3$

4. $\lim_{x \rightarrow -1^-} \frac{|x+1|}{x+1} =$
 - A. 1
 - B. -1
 - C. 0
 - D. $-\infty$
 - E. Does not exist

5. The function $f(x) = \begin{cases} -x-1 & \text{if } x < -1 \\ 0 & \text{if } -1 \leq x \leq 1 \\ x & \text{if } x > 1 \end{cases}$ is
 - A. continuous everywhere
 - B. continuous at 1 but discontinuous at -1
 - C. continuous at -1 but discontinuous at 1
 - D. continuous at all points except for 1 and -1
 - E. discontinuous everywhere

6. Find the equation of the line tangent to the curve $y = x^2 + 4x + 4$ at $(1, 9)$.
 - A. $y = 9x$
 - B. $y = 6x - 15$
 - C. $y = 6x + 3$
 - D. $y = 2x + 1$
 - E. None of the above

7. If $f(3) = 2$, $f'(3) = 4$, $g(3) = 5$, and $g'(3) = 6$, then the derivative of $\frac{f(x)}{g(x)}$ at $x = 3$ is

$$\left(\frac{f}{g}\right)'(3) =$$
 - A. 0.32
 - B. $2/3$
 - C. $-8/25$
 - D. 0
 - E. Undefined

Regular problems: show all your work

8. Sketch the graphs of:

- (a) $(x - 3)^2$
- (b) $3 \cos x + 2$
- (c) $-\sin(x - \frac{\pi}{2})$
- (d) e^{-x-1}

9. Find a formula for the function whose graph is obtained from the graph of $f(x) = e^x - 1$ by

- (a) Reflecting about the y -axis.
- (b) Vertically compressing by a factor of 5 and then shifting 3 units to the left.
- (c) Reflecting about the x -axis and then shifting 2 units down.

10. Let $f(x) = 2 - x$, $g(x) = \frac{1}{x}$, $h(x) = \sqrt{x+1}$. Find the following functions and state their domains:

- (a) $g \circ f$
- (b) $f \circ h$
- (c) $g \circ h$

11. Evaluate the limits:

- (a) $\lim_{x \rightarrow 5} (7x - 25)$
- (b) $\lim_{x \rightarrow -1} \frac{x^3 + x^2}{x^2 + 3x + 2}$
- (c) $\lim_{x \rightarrow 0} \frac{3 - \sqrt{9+x}}{x}$
- (d) $\lim_{x \rightarrow 2^+} \frac{x^3 - 2}{x^2 - x - 2}$
- (e) $\lim_{x \rightarrow 2^-} \frac{x^3 - 2}{x^2 - x - 2}$
- (f) $\lim_{x \rightarrow 2} \frac{x^3 - 2}{x^2 - x - 2}$
- (g) $\lim_{x \rightarrow 0} x^4 \cos\left(\frac{1}{x}\right)$

12. Find c such that the function $f(x) = \begin{cases} cx & \text{if } x \geq 2 \\ 5 - x & \text{if } x < 2 \end{cases}$ is continuous everywhere.

13. Show that the equation $x^5 - 4x + 2 = 0$ has at least one solution in the interval $(1, 2)$.

14. Find the vertical asymptotes of $f(x) = \frac{(x+2)(3x-4)}{(x-5)(x+7)}$.

15. Differentiate the following functions:

(a) $f(x) = 7x - 3$

(b) $p(s) = s^5 - 2s^4 + 3s^3 - 4s^2 + 5s - 6$

(c) $f(t) = \frac{3t^2 - 5t + 1}{\sqrt{t}}$

(d) $g(x) = x^2 - \frac{x^3}{\sqrt[4]{x}} + \frac{3}{x}$

(e) $q(y) = \frac{y^2 + y + 1}{y + 1}$