MATH 75

Test 2

October 31, 2003

| Namo | e: | | | |
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|------|----|--|--|--|

| Problem | Value | Score |
|---------|-------|-------|
| 1 | 3 | |
| 2 | 3 | |
| 3 | 3 | |
| 4 | 3 | |
| 5 | 3 | |
| 6 | 3 | |
| 7 | 5 | |
| 8 | 5 | |
| 9 | 5 | |
| 10 | 5 | |
| 11 | 6 | |
| 12 | 6 | |
| Total | 50 | |

Multiple choice questions: circle the correct answer

1. Find the derivative of the function $f(x) = \sqrt{x^2 - 1}$.

A.
$$\frac{1}{2x\sqrt{x^2-1}}$$
 B. $\frac{1}{2\sqrt{x^2-1}}$ C. $\frac{1}{\sqrt{x^2-1}}$ D. $\frac{x}{\sqrt{x^2-1}}$ E. $2x\sqrt{x^2-1}$

B.
$$\frac{1}{2\sqrt{x^2-1}}$$

C.
$$\frac{1}{\sqrt{x^2-1}}$$

D.
$$\frac{x}{\sqrt{x^2 - 1}}$$

E.
$$2x\sqrt{x^2-1}$$

2. Find the vertical and horizontal asymptotes for the function $f(x) = \frac{x}{x^2 - 1}$.

A.
$$x = 1, x = -1$$

B.
$$x = 1, y = 0$$

A.
$$x = 1, x = -1$$
 B. $x = 1, y = 0$ **C.** $x = 0, y = 1, y = -1$ **D.** $x = 0, y = 1$

D.
$$x = 0$$
, $y = 1$

E.
$$x = 1, x = -1, y = 0$$

3. Evaluate the limit: $\lim_{x \to \infty} \frac{9 - x^2}{5 + x}$.

$$A.-\infty$$

B.
$$-5$$

$$\mathbf{C.}\ 0$$

$$\mathbf{E.} \infty$$

4. If $f(x) = \sin^2(x)$, find $f'\left(\frac{\pi}{4}\right)$.

A.
$$-2$$

B.
$$-1$$

D.
$$\frac{1}{2}$$

5. The graph of $y = 2x^3 - x^4$ has how many local maximums?

6. Find the inflection point(s) of the graph of $y = 2x^3 - x^4$.

A.
$$(0,0)$$
 only

B.
$$(1,1)$$
 only

A.
$$(0,0)$$
 only **B.** $(1,1)$ only **C.** $\left(\frac{3}{2},\frac{27}{16}\right)$ only

D.
$$(0,0)$$
 and $(1,1)$

E.
$$(0,0)$$
 and $\left(\frac{3}{2}, \frac{27}{16}\right)$

Regular problems: show all your work

7. Evaluate the limit: $\lim_{x \to -\infty} \frac{\sqrt{5x^2 + 4}}{3x + 2}$

8. Find the linear approximation of the function $f(x) = \frac{1}{x^2}$ at a = -1.

9. Find the intervals of increase and decrease of the function $f(x) = 4x^3 - 3x^2 - 1$.

10. The volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$. Air is pumped into a spherical balloon at the rate of 100 cm^3 per second. How fast is the radius of the balloon increasing when the radius is 10 cm?

11. Find the absolute maximum and minimum values of $f(x) = (x^2 - 1)^3$ on the interval [-1, 2].

12. Find an equation of the tangent line to the curve $xy^2 + 3xy = 4$ at the point (1,1).