MATH 75
Test 1 - Answers
February 23, 2004

Multiple choice questions: circle the correct answer

1. Find the domain of the function $f(x) = \dfrac{1}{\sqrt{x}}$.

   A. $x > 0$ **B.** $x \geq 0$ **C.** $x < 0$ **D.** $x \geq 1$ **E.** $x \neq 0$

2. If $f(x) = \sin x$ and $g(x) = 5x + 1$, find $(f \circ g)(x)$.

   A. $\sin x + 5x + 1$ **B.** $\sin x(5x + 1)$ **C.** $\sin(5x + 1)$ **D.** $5 \sin x + 1$

   E. None of the above

3. Find the derivative of $\dfrac{x^2}{3x + 1}$.

   A. $\dfrac{2x}{3}$ **B.** $3x^2 + 2x$ **C.** $\dfrac{3x^2 + 2x}{(3x + 1)^2}$ **D.** $-\dfrac{3x^2 + 2x}{(3x + 1)^2}$ **E.** Does not exist

4. In which of the following intervals does the function $f(x) = x^3 + x - 5$ has a root?

   A. $[-2, -1]$ **B.** $[-1, 0]$ **C.** $[0, 1]$ **D.** $[1, 2]$ **E.** Does not have a root

5. If $f(1) = -2$, $f'(1) = 3$, $g(1) = 4$, and $g'(1) = 6$, find the derivative of the product $f(x)g(x)$ at $x = 1$.

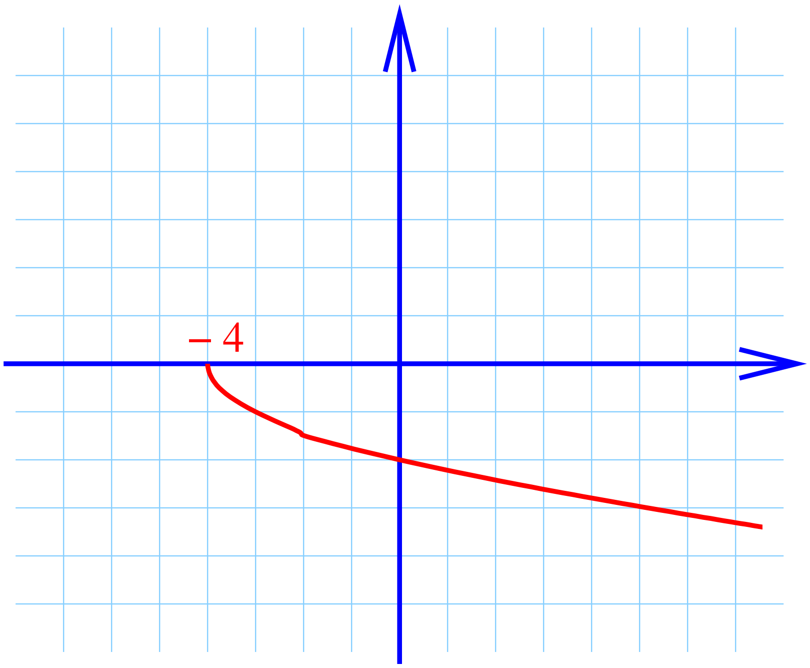
   A. 0 **B.** 9 **C.** 10 **D.** 18 **E.** -24

6. If the curve $y = e^x$ is shifted 3 units downward then the equation of the new curve is

   A. $y = e^x + 3$ **B.** $y = e^x - 3$ **C.** $y = 3 - e^x$ **D.** $y = e^{x+3}$ **E.** $y = e^{x-3}$

Regular problems: show all your work

7. Sketch the graph of $f(x) = -\sqrt{x+4}$.



8. Find an equation of the tangent line to $y = x^3 - x + 4$ at $(1, 4)$.

   The slope of the tangent line is $y'(1)$.
   $y'(x) = 3x^2 - 1$
   $y'(1) = 2$
   Then an equation is $y - 4 = 2(x-1)$, *or* $y - 4 = 2x - 2$, *or* $y = 2x + 2$.

9. Evaluate the limit: $\lim_{t \to 1} \frac{t^2 - 1}{t^2 - t}$. If the limit is infinite, determine whether it is $+\infty$ or $-\infty$.

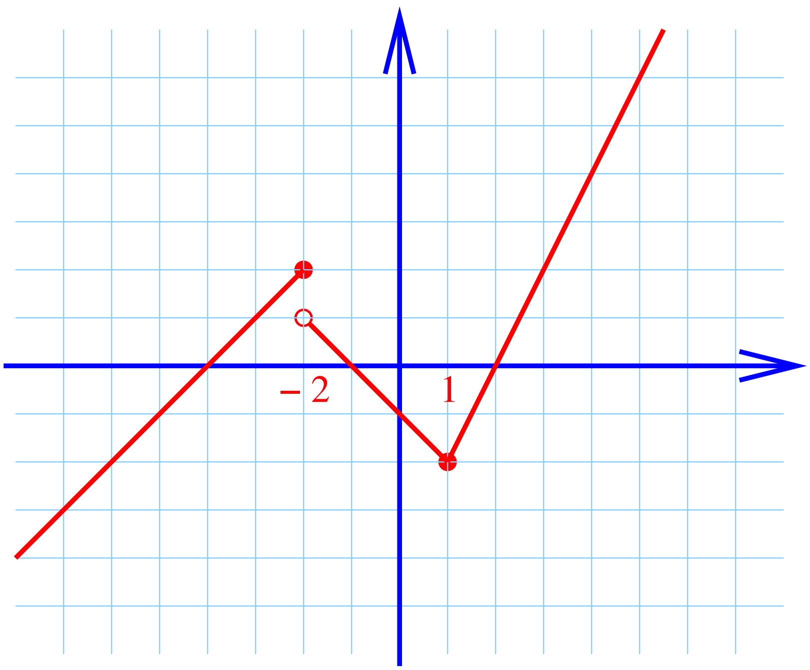
   $$\lim_{t \to 1} \frac{t^2 - 1}{t^2 - t} = \lim_{t \to 1} \frac{(t-1)(t+1)}{t(t-1)} = \lim_{t \to 1} \frac{t+1}{t} = \frac{2}{1} = 2$$

10. Evaluate the limit: $\lim_{x \to 2^-} \frac{3-x}{x^2 - 7x + 10}$. If the limit is infinite, determine whether it is $+\infty$ or $-\infty$.

    $$\lim_{x \to 2^-} \frac{3-x}{x^2 - 7x + 10} = \lim_{x \to 2^-} \frac{3-x}{(x-2)(x-5)} \left(\frac{pos.}{(small\ neg.)(neg.)} \right) = +\infty$$

11. Let $f(x) = \begin{cases} x+4 & , \text{ if } \quad x \le -2 \\ -x-1 & , \text{ if } \quad -2 < x < 1 \\ 2x-4 & , \text{ if } \quad x \ge 1 \end{cases}$.

Sketch the graph of $f(x)$.



Is $f(x)$ coninuous at -2? *No*

Is $f(x)$ continuous at 1? *Yes*

12. Find the derivative of the function $f(x) = \sqrt{x}\left(4x^2 - 2 + \frac{1}{x}\right)$.

Simplify your derivative.

First rewrite the function as $f(x) = 4x^{\frac{5}{2}} - 2x^{\frac{1}{2}} + x^{-\frac{1}{2}}$.

$f'(x) = 4\frac{5}{2}x^{\frac{3}{2}} - 2\frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{2}x^{-\frac{3}{2}} = 10x^{\frac{3}{2}} - x^{-\frac{1}{2}} - \frac{1}{2}x^{-\frac{3}{2}}$

or

$f'(x) = 10x\sqrt{x} - \frac{1}{\sqrt{x}} - \frac{1}{2x\sqrt{x}}$

(the last step is optional)