MATH 75

Test 3 - Solutions

Multiple choice questions: circle the correct answer

1. Which of the following is an antiderivative of $f(x) = \sqrt{x} + \cos x$?

A. $\dfrac{1}{2\sqrt{x}} + \sin x$ **B.** $\dfrac{x^{-1/2}}{2} - \sin x$ **(C.)** $\dfrac{2x^{3/2}}{3} + \sin x$

D. $\dfrac{x^{3/2}}{3/2} - \sin x$ **E.** $\sqrt{\dfrac{x^2}{2}} + \cos\left(\dfrac{x^2}{2}\right)$

2. $\displaystyle\int_{-2}^{5} (x+1)\,dx =$

A. -17.5 **B.** 0 **C.** $\dfrac{27}{2}$ **(D.)** $\dfrac{35}{2}$ **E.** 18.5

3. $\displaystyle\int x\sin(x^2)\,dx =$

A. $\dfrac{x}{2}\cos(x^2) + C$ **B.** $\dfrac{x}{2}\cos\dfrac{x^3}{3} + C$ **C.** $-\dfrac{x}{2}\cos\dfrac{x^3}{3}$

(D.) $-\dfrac{\cos(x^2)}{2} + C$ **E.** $\sin(x^2) + 2x^2\cos(x^2)$

4. If $f(x) = \displaystyle\int_0^x \sin(3-t)\,dt$, then $f'(x) =$

(A.) $\sin(3-x)$ **B.** $\cos(3-x)$ **C.** $-\cos(3-x)$ **D.** $x\sin(3-x)$ **E.** $-x\cos x$

5. Use Newton's Method to approximate the root of $x^5 = 5$. Let $x_1 = 1$. Find x_2.

A. 0.2 **(B.)** 1.8 **C.** 1.9 **D.** 2 **E.** 2.25

6. Find the average value of the function $f(x) = x^3 + x$ on the interval $[0, 2]$.

A. 0 **B.** 1.5 **C.** 2 **(D.)** 3 **E.** 6

Regular problems: show all your work

7. Evaluate the integral $\int_0^{\sqrt{3}} x\sqrt{x^2+1}dx$

   Make the substitution $u = x^2+1$, *then* $\frac{du}{dx} = 2x$, *therefore* $\frac{1}{2}du = xdx$, *so*
   $\int_0^{\sqrt{3}} x\sqrt{x^2+1}dx = \frac{1}{2}\int_1^4 \sqrt{u}du = \frac{1}{2}\frac{u^{3/2}}{3/2}\Big|_1^4 = \frac{u^{3/2}}{3}\Big|_1^4 = \frac{4^{3/2}}{3} - \frac{1^{3/2}}{3} = \frac{8}{3} - \frac{1}{3} = \frac{7}{3}.$

8. If $f'(x) = x^2 + \sin(x) + 1$ and $f(0) = 4$, find $f(x)$.

   An antidervative is $f(x) = \frac{x^3}{3} - \cos x + x + c$.
   Since $4 = f(0) = 0 - 1 + 0 + c$, *we have* $c = 5$, *so*
   $f(x) = \frac{x^3}{3} - \cos x + x + 5.$

9. Find the area of the region enclosed by $y = |x| - 1$ and $y = 3$

   Graph the functions. Notice that the region is a triangle with base 8 and height 4, so its area is 16. Could also use the integrals as follows. The curves intersect at $x = 4$ *and* $x = -4$. *Since* $y = x - 1$ *for positive* x *and* $y = -x - 1$ *for negative* x, *Area* $= \int_{-4}^0 (3 - (-x-1))dx + \int_0^4 (3 - (x-1))dx = \int_{-4}^0 (4+x)dx + \int_0^4 (4-x)dx = 8 + 8 = 16.$

10. Find the volume of the solid obtained by rotating about the x-axis the region under the curve $y = 4 - x^2$ and above the x-axis.

    Graph the parabola! The cross-section through a point $(x, 0)$ *and perpendicular to the* x*-axis is a disk with radius* $4 - x^2$, *so its area is* $\pi(4-x^2)^2$. *The region lies between* $x = -2$ *and* $x = 2$, *so the volume is* $\int_{-2}^2 \pi(4-x^2)^2dx = \pi\int_{-2}^2 (16 - 8x^2 + x^4)dx = \pi\left(16x - \frac{8x^3}{3} + \frac{x^5}{5}\right)\Big|_{-2}^2 = \frac{512\pi}{15}.$

11. Find the point on the line $y = 3x - 7$ closest to the point $(10, 0)$.

    The distance from a point (x, y) *to* $(10, 0)$ *is* $d = \sqrt{(x-10)^2 + (y-0)^2} = \sqrt{(x-10)^2 + y^2}$. *Since the point* (x, y) *must lie on the line* $y = 3x - 7$, *we have* $d = \sqrt{(x-10)^2 + (3x-7)^2} = \sqrt{x^2 - 20x + 100 + 9x^2 - 42x + 49} = \sqrt{10x^2 - 62x + 149}$. *We want to minimize d, but this is the same as minimizing* $D = d^2 = 10x^2 - 62x + 149$ *which is nicer. The derivative is* $D' = 20x - 62$, *and it is 0 when* $x = \frac{62}{20} = \frac{31}{10}$. *It is easy to see that this is a local minimum.* *Then* $y = 3x - 7 = \frac{93}{10} - 7 = \frac{23}{10}$.

12. Find the volume of the solid obtained by rotating about the line $x = 0$ the region under the graph of $f(x) = \frac{1}{x^3}$ between $x = 0$ and $x = 1$.

    There is a misprint in this problem (and that is why everybody got full credit for it). The given region is infinite, and we did not do integrals like this yet.

    The region is supposed to be between $x = 1$ *and* $x = 2$. *Then use cylindrical shells:*
    Volume $= \int_1^2 2\pi x\frac{1}{x^3}dx = 2\pi\int_1^2 x^{-2}dx = 2\pi\frac{x^{-1}}{-1}\Big|_1^2 = -2\pi\frac{1}{x}\Big|_1^2 = -2pi\left(\frac{1}{2} - 1\right) = \pi.$

- Good luck on the final!
- Have a nice summer break!