MATH 76

Solutions to practice problems for Final Exam

Graphs will be given on a separate sheet.

1. Sketch the graphs of the following functions.
   (a) $f(x) = 2^x - 3$
   (b) $g(x) = -\dfrac{e^{-x}}{2} + 1$
   (c) $h(x) = \ln(1 + x)$
   (d) $y = \log_5(x - 1)$
2. Find the exact value of each expression.
   (a) $\log_2 32 = \log_2 2^5 = 5$
   (b) $\log_{10} 2 + \log_{10} 50 = \log_{10} 2 \cdot 50 = \log_{10} 10^2 = 2$
   (c) $\log_4 8 = \log_4 4^{\frac{3}{2}} = \frac{3}{2}$
   (d) $2^{\log_2 6 - \log_2 3} = 2^{\log_2 \frac{6}{3}} = 2^{\log_2 2} = 2^1 = 1$
   (e) $\arcsin\left(-\dfrac{1}{2}\right) = -\dfrac{\pi}{6}$
   (f) $\arctan(\sqrt{3}) = \frac{\pi}{3}$
3. Solve the following equations.
   (a) $\ln(2x - 1) = 3$
   $2x - 1 = e^3$
   $x = \dfrac{e^3 + 1}{2}$
   (b) $2^{x-5} = 3$
   $x - 5 = \log_2 3$
   $x = \log_2 3 + 5$
   (c) $\ln x + \ln(x - 1) = 1$
   $\ln(x(x - 1)) = 1$
   $x(x - 1) = e^x$
   $x^2 - x - e^x = 0$
   $x = \dfrac{1 \pm \sqrt{1 + 4e}}{2}$
   The root $x = \dfrac{1 - \sqrt{1 + 4e}}{2}$ is negative, so $\ln \dfrac{1 \pm \sqrt{1 + 4e}}{2}$ is undefined.
   Answer: $x = \dfrac{1 + \sqrt{1 + 4e}}{2}$
   (d) $\ln(\ln(x + 1)) = 1$
   $\ln(x + 1) = e$
   $x + 1 = e^e$

$x = e^e - 1$

4. Write partial fraction decompositions of the following functions.

   (a) $\dfrac{x+1}{x^2-12x+35} = \dfrac{A}{x-5} + \dfrac{B}{x-7}$
   $x + 1 = A(x-7) + B(x-5)$
   $x + 1 = (A+B)x + (-7A - 5B)$
   $A + B = 1$ and $-7A - 7B = 1$
   $B = 1 - A$
   $-7A - 5(1-A) = 1$
   $-7A - 5 + 5A = 1$
   $-2A = 6$
   $A = -3$
   $B = 1 - A = 4$
   So $\dfrac{x+1}{x^2-12x+35} = \dfrac{-3}{x-5} + \dfrac{4}{x-7}$

   (b) $\dfrac{x^3+2}{x^2+2x+1}$
   Long division gives $x^3 + 2 = (x-2)(x^2+2x+1) + (3x+4)$
   So $\dfrac{x^3+2}{x^2+2x+1} = x - 2 + \dfrac{3x+4}{x^2+2x+1} = x - 2 + \dfrac{3x+4}{(x+1)^2}$
   $\dfrac{3x+4}{(x+1)^2} = \dfrac{A}{x+1} + \dfrac{B}{(x+1)^2}$
   $3x + 4 = A(x+1) + B$
   $3x + 4 = Ax + (A+B)$
   $A = 3$ and $A + B = 4$
   $B = 1$
   Therefore $\dfrac{x^3+2}{x^2+2x+1} = x - 2 + \dfrac{3}{x+1} + \dfrac{1}{(x+1)^2}$

   (c) $\dfrac{x^4-5}{x^3+x}$
   Long division gives $x^4 - 5 = x(x^3+x) - x^2 - 5$
   So $\dfrac{x^4-5}{x^3+x} = x + \dfrac{-x^2-5}{x^3+x} = x + \dfrac{-x^2-5}{x(x^2+1)}$
   $\dfrac{-x^2-5}{x(x^2+1)} = \dfrac{A}{x} + \dfrac{Bx+C}{x^2+1}$
   $-x^2 - 5 = A(x^2+1) + (Bx+C)x$
   $-x^2 - 5 = (A+B)x^2 + Cx + A$
   $A + B = -1$, $C = 0$, $A = -5$
   $B = -4$
   Therefore $\dfrac{x^4-5}{x^3+x} = x + \dfrac{-5}{x} + \dfrac{-4x}{x^2+1}$

   (d) $\dfrac{1}{x^3(x+1)(x^2+2)(x^2+3)^2} = \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x^3} + \dfrac{D}{x+1} + \dfrac{Ex+F}{x^2+2} + \dfrac{Gx+H}{x^2+3} + \dfrac{Ix+J}{(x^2+3)^2}$
   The coefficients can be found in the same way as above bu the calculation is very long.

5. Sketch the following curves.

(a) $x = 2t,\ y = 3t^2$

(b) $x = 2\cos(t),\ y = 3\sin(t)$

(c) $r = -2\theta$

(d) $r = \cos(5\theta)$

(e) $x - y^2 - 2y - 3 = 0$

(f) $x^2 + y^2 - 2y - 3 = 0$

(g) $4x^2 + y^2 - 2y - 3 = 0$

(h) $4x^2 - y^2 - 2y - 3 = 0$